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LE SENS DE L'ORIENTATION : IMPACT DE L'ASYMÉTRIE ROUTIÈRE SUR LA STRUCTURE EN "PETIT-MONDE" DES RÉSEAUX DE RUES

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TABLE DES MATIÈRES

LISTE DES FIGURES

LISTE DES TABLEAUX

LISTE DES ALGORITHMES

RÉSUMÉ

Un nombre grandissant de recherches en cognition spatiale démontre que l'orientation et le mouvement urbains sont déterminés par une quantité restreinte d'informations, relevant davantage de la topologie que de la géométrie métrique. Une nouvelle forme de modélisation routière, dite à base de rues, s'avère d'ailleurs particulièrement bien adaptée à la représentation de cette expérience cognitive urbaine. Dans ces modèles, la rue est conçue comme un ensemble ordonné de segments adjacents à sériation odonymique ou angulaire : dans le premier cas, une rue est formée de segments adjacents portant le même nom ; dans le second, une rue est formée de segments adjacents caractérisés par des rapports d'incidence inférieurs à un seuil d'angularité donné. Structurellement, les réseaux de rues diffèrent radicalement des représentations géométriques traditionnelles. Il a notamment été démontré que les réseaux routiers urbains, jusqu'alors considérés comme l'un des rares types de réseaux réels ne présentant pas de propriétés de réseaux en petit monde, s'avèrent au contraire petits-mondains lorsque représentés sous forme de réseaux de rues. L'impact de ces résultats est toutefois amoindri par le fait que les modèles à base de rues ne tiennent pas compte de l'unidirectionnalité de certaines routes, dites "à sens unique". Au moyen de l'utilisation de l'indice ω, une mesure de "petite mondanité" développée dans le cadre de l'analyse des structures cérébrales, la présente recherche vise à évaluer l'impact de l'asymétrie routière sur le caractère petit mondain des modèles à base de rues. L'analyse des réseaux de rues de trois quartiers londoniens (Barnsbury, Clerkenwell, Kensington) entreprise ici permet de démontrer que la prise en compte de l'asymétrie routière dans la construction des modèles à base de rues permet non seulement d'en mieux souligner la structure petit-mondaine, mais aussi et surtout la structure presque parfaite des réseaux de rues odonymiques à cet égard. À la lumière des piètres performances humaines en matière de navigation et d'orientation urbaines, cette petite mondanité exceptionnelle des réseaux odonymiques montre bien l'importance cognitive et navigationnelle de la lexicalisation des réseaux routiers, essentielle à la vie et au mouvement urbains.

INTRODUCTION

Much of what lies on Earth's surface and constitutes our living environment can be defined as geographic space, that is, the space "that contains objects that we humans do not think of being manipulable objects" (Egenhofer and Mark, 1995, p.4):

> Geographic space is larger than a molecule, larger than a computer chip, larger than a table-top. Its objects are different from an atom, a microscopic bacterium, the pen in your hand, the engine that drives your car. Geographic space may be a hotel with its many rooms, hallways, floors, etc. Geographic space may be Vienna, with its streets, buildings, parks, and people. Geographic space may be Europe with mountains, lakes and rivers, transportation systems, political subdivisions, cultural variations, and so on (Egenhofer and Mark, 1995, p.4).

In such large-scale spaces, objects, phenomena and activities belong to what could be called "the supra-haptic realm", that is, the domain of all things that are beyond grasp, that cannot be manipulated, handled or crafted by unaided organisms, whether they be animal or human. In this steady, resilient and seemingly monolithic environment, mobility is the key to survival: foraging, mating, predating, evading, hunting, gathering, nurturing, migrating, exploring or even perceiving are all locomotive activities that rest on the ability to get around efficiently and quite often on a moment's notice. Despite countless technological breakthroughs and quality of life enhancements, modern life hasn't reduced in any way this need for mobility, nor its omnipresence:

> Our daily life is filled with activities traveling from one location to another. Every morning, we walk from the bedroom to the bathroom, from there to the kitchen, etc. Later, we drive from home to work or take the bus into town. We stroll through the city we live in or drive to places far away to visit friends (Werner et al., 2000, p.295)

From a cognitive point of view, such activities are also quite distinct from those that pertain to smaller-scale, table-top spaces, in that they rest on organisms' wayfinding abilities, that is, on their capacity to purposefully determine or follow a route between an origin and a destination that cannot be directly perceived (Golledge, 1999; Raubal, 2008). As "purposeful mobility" (Passini, 1996, p.322), wayfinding involves four different stages (Downs and Stea, 1977).

- Orientation: Determination of one's location relative to nearby objects and the destination.
- Route Decision: Selection of a route to get to the destination.
- Route Monitoring: Making sure the selected route leads to the destination.
- Destination Recognition

From an information-processing perspective, these different activities pertaining to wayfinding are extremely complex and demanding. Despite this, most animal species have extremely advanced wayfinding abilities (Schöne and Strausfeld, 1984; Waterman, 1989; Gallistel, 1990; McNaughton et al., 1990).

> Research in behavioral ecology, experimental psychology, and cognitive neuroscience provides evidence that a wide variety of animal species, from insect to mammals, maintain an exquisitely precise sense of their own position in relation to significant places in the environment. Animals update their spatial representation as they move around, and they draw on those representations in navigating through the layout, reorienting themselves, and locating objects (Spelke and Tsivkin, 2001, p.72).

In this regard, ants and bees constitute evocative examples. When leaving the anthill in search for food, the Sahara desert Ant (*cataglyphis fortis*) follows a long and sinuous path several meters long, then returns directly to the nest when its objective if fulfilled, following a straight-line path. This spatial prowess is not due to information emitted by other ants but is rather the product of a sophisticated internal navigation system: if the ant is picked up at the food source and relocated elsewhere, the return trajectory it follows corresponds exactly, in terms of both distance and direction, to the path that would have allowed it to return directly to the nest (Collett, 1998; Collett and Collett, 2000). The dances made by bees in order to specify the location of a nearby food source constitute another good example, distance and direction information being respectively communicated by rotation speed and inclination degree to the vertical axis, with a precision or more or less 3 degrees (Frisch, 1967; Menzel et al., 1998; Wehner et al., 1996).

Recent research done in non-human wayfinding has shown that such wayfinding prowesses are often due to sophisticated sensory mechanisms, allowing for the detection of directional information on the basis of various gradients, magnetic fields (in the case of migratory birds) or weak electrical fields (rays and sharks) (Waterman, 1989; Hughes, 1999). As for the aforementioned ants and bees, both are "equipped", like many arthropods, of photic polarity detectors allowing them to use the sun as compass: by transporting bees by night and plane across different time zones, it was discovered that their orientation sense was disturbed, their first flight after getting out of the plane being oriented according to the location where the the sun should have been based on their inner compass (Landau, 2002). Certain bird species, for example garden warblers (Wiltschko and Wiltschko, 1995), pigeons (Wilzeck et al., 2010), and robins (Wiltschko et al., 2002; Hein et al., 2011), have magnetic compasses in both eyes, allowing for optimal migratory orientation. The polarity compass of certain northern hemisphere birds even react to the apparent motion of stars: in a sophisticated experiment (Emlen, 1967a,b), indigo buntings, which normally use Polaris to orient themselves, were raised in a planetarium in which the celestial archway revolved around Beetlejuice instead, which resulted in the modification of their migratory direction.

In fact, both biological and experimental evidence regarding the spatial expertise of animals is so compelling and ubiquitous that it might seem possible to assert that "there is no creature so lowly that it does not know, at all times, where it is" (Gallistel, 1990). However, experiments and even casual experience show that humans are an exception to this rule:

> Many people living in modern, technological societies appear to retain very little sense of their position or orientation, or of the egocentric directions of significant objects and places, as they move. Perhaps as a consequence, people often navigate on strikingly inefficient paths, even through familiar environments (Spelke and Tsivkin, 2001, p.72).

Indeed, studies have shown that humans often fail to properly negotiate return paths (Gillner and Mallot, 1997) or recollect metric information (Byrne, 1979; Moar and Bower, 1983; Moar and Carleton, 1982; Tversky, 1981, 1998). Also, sketch maps that research participants are asked to produce poorly represent the actual Euclidean distances involved (Devlin, 2012; McNamara et al., 1984; Moeser, 1988). Moreover, various studies show that directions between objects that belong to different regions are frequently distorted, while distance estimations across barriers or region boundaries are exaggerated as compared to distances that do not cross boundaries (Kosslyn et al., 1974; Cohen et al., 1978; Stevens and Coupe, 1978; Thorndyke, 1981; Newcombe and Liben, 1982; McNamara, 1986; McNamara et al., 1989; McNamara and LeSueur, 1989). Wayfinding experiments in virtual environments with landmark inconsistencies have also shown that human subjects did not perceive or report inconsistencies in spatial layout (Gillner and Mallot, 1998; Mallot and Gillner, 2000; Steck and Mallot, 2000).

One possible explanation for this lack of wayfinding competence is that "we derive from a primate line that displays no outstanding navigational abilities" (Levinson, 2003, p.223).

> Overall, there is no evidence in the primate order of the quite striking spatial

> abilities in other animals, and there is no evidence that the larger-brained species are any more resourceful navigators than smaller-brained monkeys (Levinson, 2003, p.222-223).

Of course, the poor wayfinding skills of humans and primates in general might partly be attributed to the fact that they missed out "on the bonanza of special-purpose biological navigation aids" (Levinson, 2003, p.223) that most other animal species were granted with. As regards to other primates, these sensorial shortcomings are no really big deal, however, given their sedentary lifestyle: besides humans, no primates travel more than 10 km per day over areas larger than 50 km^2 (Tomasello and Call, 1997, p.28). However, for a species as mobile as ours, such biological and cognitive insufficiencies certainly had to be compensated for in one way or another, otherwise bare survival would have been strongly compromised. This adaptive challenge, far from being an evolutionary dead end, allows on the contrary to emphasize the importance and impact of one of nature's most overlooked phenomena, which might be at the source of humans' main adaptive asset: niche construction.

0.1 NICHE CONSTRUCTION: ECOLOGICAL, EVOLUTIONARY, CUMULATIVE AND DOWNSTREAM EFFECTS

From a thermodynamic point of view, organisms are highly improbable, open and out-of-equilibrium systems. In order to survive and reproduce, each living being has to maintain a strict energy budget through relentless matter and energy exchanges with their environment:

> To stay alive, organisms must be active as well as reactive. Organisms must gain resources from their external environments by genetically informed, or possibly brain-informed, fuel-consuming, nonrandom work. They must return detritus to their environments, and they must choose and perturb specific components of their environments when they do so (Odling-Smee, 2010, p.179).

Given this incessant activity, it would be misleading to assert that organisms cope with their environment. In fact, all organisms, through their metabolisms and behavior, partly construct and regulate their own niche at their expense of their environment, an activity which often has ecological, selective and evolutionary consequences.

> A common picture of evolution by natural selection sees it as a process through which organisms change so that they become better adapted to their environment. However, agents do not merely respond to the challenges their environment pose. They modify their environments, filtering and transforming the action of the environment on their bodies. A beaver, in making a dam, engineers a stream, increasing both the size of its safe refuge and reducing its seasonal variability. Beavers, like many other animals, are ecological engineers. They act to modify the physical challenges posed by their environment. Nests, burrows and other shelters reduce the impacts of adverse weather and of other agents. Animals also modify their exposure to biological risks. Hygienic behavior reduces the impact of disease. Intensive grooming; moving to new roosts; using a "latrine burrow"; disposing excrement in faecal sacs; these all improve an animal's prospects of avoiding disease. So many organisms are like the beaver; they partially construct their own niches. They are ecological engineers, and (...) niche construction is often of great evolutionary significance, transforming the effects of natural selection on both the ecological engineers and their descendants. Ecological engineering is visible to selection, for such alterations often have fitness effects that are stable across generations. So niche-constructing behavior itself evolves (Sterelny, 2004, p.239).

Often, the benefits of such regulatory activities cannot be simply reduced to material and energetic considerations, as ecological engineering sometimes have huge consequences as regards to information processing. Such modifications to the informational character of an organism's environment, pertaining to what has been called epistemic engineering (Sterelny, 2003), "are a common feature of animal life"(Sterelny, 2003, p.239): agents act to modify their information world in order to simplify their cognitive tasks, just as they act to modify their physical world in order to optimize their energetic budget. From this epistemic engineering perspective, geographic space constitutes a crucial commodity:

> An animal that marks its territorial boundaries with scent unloads information into the world, for the animal no longer has to remember the location of those boundaries. A memory problem is now solved by sensory mechanisms that the animal has available on-line and which work automatically. More routinely, animals move through their environment to improve their epistemic situation. Thus, potential prey quite often accept the risks involved in inspecting predators. They probe their environment at some risk to get the information they need (Sterelny, 2003, p.148).

As with ecological engineering, epistemic engineering often have consequences that reach beyond the engineer's own niche: it can modify the niche of others species, the niche of the engineer's descendants or even the niche of organisms of the same species as the engineer's. Both ecological and epistemic engineering can also be cumulative and have what Sterelny calls "downstream consequences", each new engineer adding to the improvements made by its predecessors. However, such activities have most of the time a local impact and seldom evolve:

> A group of rabbits in a warren constructs their tunnel system cumulatively over a number of different rabbit generations. The tunnel system becomes more extensive and acquires more escape holes, but these changes will not ramify beyond this tunnel system. The rabbits have modified their burrows multi-generationally, but not modified the way they make burrows multi-generationally. The latter is a change which might have less local consequences"(Sterelny, 2003, p.149).

In fact, ecological and epistemic engineering that can both cumulate and affect a whole population, that is, the totality of organisms of a given species, is a human prerogative.

> The domestication of plants and animals - a process of incalculable significance in human history (Diamond, 1998) - was a cumulative process of this kind. Before domestication, foragers cumulatively modified the biological profile of their habitat. For example, as hominid weapons and cooperation improved, dangerous predators became rare and wary. Indeed, some became terminally rare. Furthermore, many species that survived hu-

> man impact did so by changing behaviorally: the huge herds of buffalo in North America were a response to human hunting pressure (Flannery, 2001). Others adjust their life-history patterns, with earlier reproduction and a smaller body size (Flannery, 1994, 2001). In short, niche construction is an important aspect of the relations between organisms and their environment. Downstream niche construction is likewise common. But *cumulative downstream niche construction* is a hominid specialty (Sterelny, 2003, p.150)

As regards to wayfinding, this downstream cumulative niche construction is precisely what allows humans to compensate for their lack of innate wayfinding capacities and poor spatial sense. In order to reduce and overcome the difficulties of wayfinding, humans greatly dampened the spatial complexity of their environment by reducing both physical and perceived degrees of freedom through the construction of roads and road networks. Indeed, clearing land in order to make a road from one location to another facilitates movement not only by removing potential obstacles from the way or allowing for heavier, denser traffic, but also by indicating the way to go, thus dispensing with complex spatial information-processing operations such as calculating distances and directions. While traveling on a road, cognitive complexity arises only at decision points: until the next road crossing or even until arrival, wayfinding problems are reduced to a simple cognitive task: "keep moving forward", thus allowing travelers to focus on other, more cognitively demanding tasks.

This form of epistemic engineering relates to what psychologist David Kirsh calls " jigging" the environment, that is, structuring or preparing the environment in order to decrease its variability and adapt it to an organism's existing skills and capacities (Kirsh, 1995). In the case of road networks, the environment is jigged physically, "by planting physical impediments or constraints in the environment to reduce the *physical* degrees of freedom an agent actually has" (Kirsh, 1995, p.38). By reducing the number of spatial trajectories that can be taken and by making direction and distance computation less necessary and crucial to navigation, road networks can thus be considered as much epistemic as ecological artifacts, thus representing an exemplary case of "intelligent use of space" (Kirsh, 1995, p.31).

0.2 EPISTEMIC ROAD ENGINEERING AS AN "INTELLIGENT USE OF SPACE"

Road-making is certainly no human prerogative, as "the first pathways to cross the earth were created by animals pushing aside vegetation and pouring the earth with their feet" (Lay, 1992, p.5). A 1802 report to the British House of Lords written by Thomas Telford is testimony to that claim: discussing about the north of Scotland, Telford noted in this report that "previous to the year 1742, the Roads were merely the Tracks of Black Cattle and Horses" (Telford (1802), cited in Lay, 1992, p.5). In North America, the pioneering wanderings of buffalo herds have exerted a strong influence on the subsequent construction of road networks: "the buffalo, because of his sagacious selection of the most sure and direct courses, has influenced the routes of trade and travel of the white race as much, possibly, as he influenced the course of red men in earlier days" ((Hulbert, 1971), cited in Lay, 1992, p.5). However, it has often been remarked that animal paths are too inconsistent to have carved the first roads (Forbes, 1934; Roe, 1929, 1939). While claims relating to the animal origins of road networks are certainly debatable, it is however clear that animal paths are as old as animal mobility itself and that both their quality, durability, and probable use by humans - ultimately depend on local terrain and vegetation conditions:

> Difficult terrain or dense vegetation in fertile areas would certainly have required narrow and specific animal ways. Subsequently, animal husbandry and the the cultivation of crops would have reinforced the need for ways to either permit animals to access pasture land or prevent them from doing so (Taylor, 1979). On the other hand, in open territory the ways would have been much broader, although the North American experience clearly shows that they still provided convenient paths for many travelers (Lay, 1992, p.6).

Paths pushed and trampled by animals couldn't however compare to the sophistication, durability and effectiveness of manufactured, human-made roads. In fact, road-making certainly counts among the oldest and most fundamental ecological engineering feats of humankind. The first known pathways used by human travelers are more than 12,000 years old. Early African explorers reported having been able to cross the entire continent from east to west by following village-to-village footpaths (Gregory, 1931). In America, road networks were well developed even before Europeans arrived in North America (Labatut and Lane, 1950; Rose, 1950). As proof to that claim, United States official documents published as late as 1808 were still referring to roads that did not follow native trails as artificial roads (Gallatin, 1808, p.7).

As ancient as ubiquitous, manufactured paths quickly became of the outmost social and economic necessity, as the growth of communities "created the first major need for organized travel, the catalysts being the requirements of trade and the collection of superimposed taxation" (Lay, 1992). The emergence of urban communities is also closely connected to the spreading of road networks: "urban societies appeared in Mesopotamia around 4000 B.C. and, not coincidentally, villages with street paving also date from about this time" (Lay, 1992, p.12). While the first incentive for the creation of cities has been joint defense, trade soon became the dominant factor of urban development. For urban road networks, this shift in priorities has resulted in major structural modifications: while the chaotic and narrow configurations of early street systems were a useful tool against invaders (Mumford, 1961), they soon became a serious impediment to the transportation of goods, a reality which quickly illustrated the crucial importance of efficient road network infrastructures for trade and manufacture facilitation.

Given the importance of these commercial imperatives, the design of efficient urban road systems soon became a source of speculation, experimentation, design and engineering in the most economically active regions of the world. Geometrically ordered configurations of wider, straight streets appeared in Middle Eastern cities around 2000 B.C. (Lay, 1992; Haverfield, 1913). Urban road network designing also became a true profession, as Hippodamus became the first recognized town planner, advocating a method which later came to be known as Milesian planning.

> His planning approach was not responsive to the locale; rather, it was rectilinear and geometric and reliant on a checkerboard regularity and rectangu-

> larity to lead inevitably to a grid-like street network. Hippodamus saw this process as a triumph of reason over the wanton riot of nature (Lay, 1992, p.13)

Such grid-like configurations of street networks were not new, as similar, thousand years old structures already existed at the time (Lay, 1992). However, the importance of Hippodamus is that "he formalized the practice of rigidly applying absolute principles while paying scant attention to local topography - a tradition boldly followed by many subsequent town planners"(Lay, 1992, p.13).

As regards to the link previously established between road networks and epistemic engineering, this enduring precedence of orthogonality, regularity and legibility over topographical fitness is very telling: while the ubiquity and ancientness of road networks hints at their necessity from a human niche construction perspective, the question remains open as to whether the benefits of these environment-modifying activities essentially served energetic or informational purposes. In other words, is the main objective of road-making the minimization of energetic expenditures or the alleviation of human cognitive workload as regards to wayfinding? In the case of urban areas, it seems as though the simplification of the urban layout through the construction of orthogonal, grid-like road networks aims to compensate for the relative lack of wayfinding skills of their users, not to simply fit the geometric shape of the geographic layout. In this sense, good urban design would pertain as much to epistemic engineering and wayfinding than ecological engineering and transportation.

> If the part of our cognitive apparatus that allows us to predict other's whereabouts, to act socially, and to locate ourselves with respect to others in an intentional way is exosomatic - located in an environment constructed largely by others - we are disabled if that apparatus is poorly structured. This is the effect of unintelligible space. By making it impossible for individuals to act intentionally, we remove autonomy as surely as if we tampered directly with the brain (Penn, 2003, p.62)

In this regard, the results obtained by what is here called "street-based models" are evocative. As their definition in 1.1.4 will show, such models seek to represent the way road networks are experienced by their users instead of focusing on their geometric layout. Given this aim, such models focus on whole streets, defined on the basis of odonyms (street names) or deflection angles and overall street topology instead of individual road segments, metric distances, directions and global road geometry. Structurally, these street-based models are radically different from traditional models, as research has shown that network topology is as effective for local navigation as it is for longer trips inside the network. This certainly isn't the case of geometric representations of road networks, which contain too much information for effective navigation from a human point of view. As testimony to this contrast in structural efficiency, network analysis has shown that, whereas traditional road network cannot be likened to small-world networks, known for their well-balanced structure as regards to both local and global efficiency, opposite results have been obtained for street-based models. These considerations certainly add weight to the thesis of the epistemic engineering of road networks. However, street-based models do not take into account road asymmetry, that is, the fact that most urban road networks have a non negligible number of unidirectional, one-way roads. In order to address this discrepancy and to properly evaluate the well-foundedness of the epistemological road engineering thesis, the objective of the present research is to investigate this epistemic-ecological dilemma in the context of the instauration of one-ways in urban road networks.

0.3 ONE-WAYS: EPISTEMIC OR ECOLOGICAL ARTIFACTS?

In order to help alleviate congestion problems in the Roman capital, Julius Caesar implanted the first one-way streets in 45 B.C. The proliferation of one-way streets is however closely linked to the emergence of motorized transportation in the last century (Lay, 1992). From a global, structural perspective, one-ways radically modify the urban landscape, by introducing asymmetry in road networks through the conversion of bidirectional roads that allow traffic in both directions into unidirectional road sections. Given such alterations, urban road networks in which one-ways are introduced lose their symmetric nature and become non-symmetric, as the roads they contain are neither exclusively symmetric (some of them are one-way roads) nor exclusively asymmetric (some of them are two-way roads).

From a transportation and ecological engineering perspective, the relevance and importance of such initiatives is obvious, as the structural modifications caused by the conversion of two-way into one-way streets are aimed at decreasing local or global traffic (Tomko et al., 2008). However, the question remains as to whether the introduction of one-way street also befits cognitive needs. After all, extending sidewalks or installing road blockers or poles in order to prevent cars from taking a one-way street in reverse direction does reduce effective degrees of freedom. As for one-ways signs, they exemplify a second form of environment jigging: according to David Kirsh, agents jig their environment not only physically, but also informationally, by seeding attention-getting and information-rich objects or structures in the environment in order to reduce perceived degrees of freedom (Kirsh, 1995). According to this definition, one-way signs, by indicating both the unidirectional nature of certain roads as well as their direction, constitute informational jigs of the outmost importance. Thus, directional restrictions caused by the conversion of two-way roads into one-ways reduce both perceived and physical degrees of freedom, thus limiting the number of possible routes for given origins and destinations and simplifying wayfinding tasks. In this sense, road direction is intimately linked to wayfinding and spatial cognition. Taking this line of thought further, studies have shown that road direction does have an impact on cognition:

> As Hermann et al. (1995) were able to show, retrieval of information about objects along a route is easier in the direction of the route than in the opposite direction. If an object is briefly presented as a cue it is thus easier to recognize the name of the subsequent object than the preceding object on the route. This so-called *route direction effect* (*Richtungseffekt*) does not occur if the objects are just *displayed* in the same sequence as when traveling the path or when the optical flow is revised while learning the route (Hermann et al., 1995; Schweizer et al., 1998). These results indicate that spatial relations are encoded in a direction-specific way, and that the relation between two objects or places A and B is not necessarily the same as the relation between B and A (Werner et al., 2000, p.298).

Such considerations regarding route direction effect are all the most crucial, given that the above street-based models do not take into account the non-symmetric nature of urban road networks (Jiang, personal communication). Indeed and as surprising as it may seem, all street-based models rely on the use of undirected graphs, which results in the loss of all information relative to road direction. Given that this fact certainly dampens the impact and significance of these models for road-network modeling and human epistemic engineering and wayfinding in general, the present research aims to address this massive discrepancy by analyzing the impact of the non-symmetry of road networks on their structural efficiency, as revealed by the small-worldness of street-based models.

0.4 RESEARCH OUTLINE

In the first chapter, concepts and methods pertaining to both small-worlds and street-based modeling of road networks are presented. In particular, it will be shown how, from mere anecdotal, ludic or literary evocations, small-worlds became the object of intense sociological experimentation, following the pioneering studies of de Sola Pool, Kochen and Milgram, and then triggered the emergence of transdisciplinary network research through the development and dissemination of Watts and Strogatz's small-world network model. In the case of road networks, it will be shown that, to the contrary of most transportation systems, traditional representations of road networks do not present small-world-related properties, due to the that that geographic and metric embedding poses severe constraints on the number of both hubs and shortcuts. However, a new street-based and user-centered modeling technique for road networks has been devised in the last decade, technique which has brought closer small-worlds and road networks. Following a description of this new modeling approach, the chapter then shows how the overlooking of road asymmetry by these models proves detrimental to their impact and validity, and how the impact of road asymmetry on street networks can be properly assessed by means of the ω metric, a structural indicator designed in brain network research.

As regards to the second chapter, it constitutes a sort of formal introduction to this research. After a short presentation of the logical connectives and set-theoretic concepts in sections 2.1 and 2.2 respectively, relations and operations used in the research procedures, most graph concepts and operations are defined in section 2.3. Following this, vector databases and vector-based modeling of road networks, fundamental to the data collection and preprocessing as well as to the graph generation stages of this research, are presented.

In the third chapter, the overall methodological framework of the present research is presented. This chapter is of the outmost importance since, to our knowledge, no research has so far presented a full formal framework for the generation of street-based models; while algorithmic descriptions of street recovery procedures have already been presented in the literature, the formal, graph-theoretic concepts and operations underlying these procedures are lacking. The following research methodology is mainly based on the generation and analysis of symmetric and non-symmetric street networks. The construction of these networks follow a four-step process: following a data collection and preprocessing phase, the vector maps of the three London neighbourhoods under study (Barnsbury, Clerkenwell, Kensington) are converted into road networks. These geometrical models are planar straight line graphs in which angles and distances are respectively kept identical and proportional to those of the corresponding vector maps. Two types of road networks are generated, depending on road direction: if the latter is taken into account, non-symmetric road networks are created through generation of mixed graphs; otherwise, symmetric road networks are build using undirected graphs. This procedure results in the generation of one symmetric road graph and one non-symmetric road graph for each of the three neighbourhoods, which results in a total of six different road graphs. Thirdly, in order to recover the street structure of each neighbourhood, different arc-and-edge-disjoint path partitions are built for each each symmetric and non-symmetric road network, each having distinct parameterization settings. The first partition parameter is street type: in the present research, three different street types are proposed, namely segmental streets, odonymic streets and angular streets. Segmental street paths are formed by including in the same street adjacent road sections comprised between two intersections. As for odonymic street paths, they are built by joining adjacent road sections which have the same odonym (or street name). Finally, angular street paths are formed through the merging of adjacent road sections whose deflection angle satisfy specific angularity requirements. These conditions constitute additional parameters, unique to angular path partitions. The first angular parameter, angular fitness type, refers to different requirements adjacent road sections must satisfy in order to be added to a given street path. Three such fitness types are distinguished: simple fitness, relative fitness and mutual fitness; each fitness type will be defined in section 3.3.4.3. Finally, angularity threshold constitutes the upper bound for angular street path formation: two adjacent road sections forming a deflection angle whose value is greater than the angular threshold given as input cannot be part of the same angular street path. Six different threshold values have been selected in this study: $20^o, 30^o, 40^o, 50^o, 60^o, 70^o$. In light of this, 20 different parameter configurations are possible for each neighbourhood, which results in the generation of 120 different path partitions in total. Finally, in order to evaluate the small-worldness of symmetric and non-symmetric street networks, three different types of directed graphs are generated. First of all, symmetric and non-symmetric street networks are obtained by generating intersection graphs of all path partitions of road networks; their respective average clustering coefficient and characteristic path length are recorded for further ω score analysis. Following this, 10 isosequential (with identical in-degree and out-degree sequence) random graphs and 10 isosequential highly-clustered graphs are generated on the basis of the in-degree and out-degree sequences of each street network; then, the mean characteristic path length and mean average clustering coefficient of those isosequential graphs are respectively recorded.

Following generation of all these directed graphs, ω scores, analysis and comparison for all street networks are shown in chapter IV. As the results show, taking into account road asymmetry better highlights not only the small-world nature of street networks, but also the exceptionally efficient structure of odonymic (street names-based) networks, a discovery which leads to interesting insights regarding the design of navigable and intelligent urban spaces.

CHAPTER I

SMALL-WORLD AND THE NAVIGABILITY OF ROAD NETWORKS

Of paramount importance to complex networks theory, small-world-related research and analysis represent the investigative and scientific response to interrogations stemming from the discovery that people who don't seem to have anything in common are often connected through unexpected shared acquaintances.

> Almost all of us have had the experience of encountering someone far from home, who, to our surprise, turns out to share a mutual acquaintance with us. This kind of experience occurs with sufficient frequency so that our language even provides a cliché to be uttered at the appropriate moment of recognizing mutual acquaintances. We say, "My it's a small world" (Milgram, 1967, p. 61).

Such startling and counterintuitive anecdotal experiences proved conducive to narrative explorations, experimentations and depictions. In the second half of the last century, however, the "small-world effect" became the focus of original sociological experiments as well as a hot topic in social network research. The development and dissemination of the Watts-Strogatz model however caused a giant research boom, and a new science of networks emerged of the transdisciplinary effervescence, interest and breakthroughs surrounding small-world-related research. In this regard, research related to road networks yielded negative results, until the development of street-based modeling has led to the discovery of small-world properties in street networks. Such discovery is however hampered by the fact that street-based models have so far overlooked road asymmetry. In light of this, the present research attempts to evaluate the structural impact of road asymmetry on the small-worldness of street networks, using a small-worldness measure recently developed in brain network research. The present chapter aims to explain in greater detail the theoretical background of the present research as well as highlight its main objectives.

1.1 SMALL-WORLDS, BETWEEN SHORTCUTS AND HUBS

The earliest and best-known literary depiction of the "small world effect" is generally attributed to the prolific Hungarian polygraph Frigyes Karinthy (1887-1938). In a short story entitled "Chain-Links" (*Láncszemek*) and published in 1929, the authors raises, in a fictional and literary fashion deprived of any scientific intent, issues of utmost relevance to modern network theory. In a way reminiscent of what his idol Jules Verne alluded to 50 years earlier in "Around the World in Eighty Days", Karinthy argued that human exploration, demolition of geographical boundaries and technological developments has made the world smaller. Whereas this shortening of distances is physical in Verne's book and has proved through the circumnavigation of the globe, Karinthy's argument relates rather to the social realm and is adduced in the form of a game devised spontaneously during a vivid but fictive discussion between imaginary participants.

> A fascinating game grew out of this discussion. One of us suggested performing the following experiment to prove that the population of the Earth is closer together now than they have ever been before. We should select any person from the 1.5 billion inhabitants of the Earth - anyone, anywhere at all. He bet us that, using no more than five individuals, one of whom is a personal acquaintance, he could contact the selected individual using nothing except the network of personal acquaintances. For example, "Look, you know Mr. X.Y., please ask him to contact his friend Mr. Q.Z., whom he knows, and so forth" (Karinthy, 2006, p. 22).

After a participant was able to "connect himself" with the Swedish author and nobel laureate Selma Lagerlöf through Béla Kehrling, a well-known athlete of the time, and the King of Sweden, the protagonist of the story then proposes to find a chain of contacts between himself and an anonymous riveter working at the Ford Motor Company, a problem he solved in four steps. And the game went on, nobody in the group needing more than five steps to reach any inhabitant of the planet through this method of acquaintance.

On a more autobiographical note, ludic experiments based on acquaintance chains have also been evoked by the author, activist and urban planning *aficionado* Jane Jacobs in her book *The Life and Death of American Cities*:

> When my sister and I first came to New York from a small city, we used to amuse ourselves with a game called Messages. The idea was to pick two wildly dissimilar individuals - say a head hunter in the Solomon Islands and a cobbler in Rock Island, Illinois - and assume that one had to get a message to the other by word of mouth: then we would each silently figure out a plausible, or at least possible, chain of persons through which the message could go. The one who could make the shortest plausible chain of messengers won (Jacobs, 1961, p.134-135).

Many similar literary excerpts and personal anecdotes contributed to make the small world phenomenon both extremely familiar to anyone, yet always surprising to witness. Despite their entertaining and evocative power, these ludic and literary prowesses were however nothing more than results of creative imagination. In order for the small-world effect to extend beyond the realm of pure fiction and anecdotal evidence and become a tangible object of scientific investigation, strong assumptions had to be made regarding the structure of social networks, assumptions whose validation could only come through the design of unprecedented methodologies.

1.1.1 THE SMALL-WORLD METHOD: "A CLUE TO SOCIAL STRUCTURE"

The small-world effect became an object of scientific interest and inquiry in the late 50s, when the preliminary results of a research on patterns in social contacts, co-led by political scientist Ithiel de Sola Pool and mathematician Manfred Kochen of IBM, started circulating informally among academic circles. For the authors, meetings such as those related above are not random or freak occurrences, but rather "a clue to social structure" (De Sola Pool and Kochen, 1978, p. 5). By assuming that the number of individuals each person in the network knows affects how far everyone is from everyone else in the network, Pool and Kochen used randomly generated graphs to make conjectures about the structure of contact networks and how such structure clusters "who might know whom" (Kadushin, 2012, p.110).

> Starting with the assumption that each person has about 1,000 acquaintances, they predict that most pairs of people on Earth can be connected via a path that goes through just two intermediate acquaintances. (...) They also consider the possibility that community groupings and social stratification within the network would affect their conclusions. But, after some laborious calculation, they conclude, apparently to their own surprise, that social strata have only a small effect on distance between individuals (Newman et al., 2006, p. 15).

However, as the authors admit themselves, their research "raises more questions than it answers" (De Sola Pool and Kochen, 1978, p. 5). In an attempt to test Pool and Kochens's surprising conjectures about path lengths between individuals in social networks, Harvard sociologist Stanley Milgram devised a new kind of experiment, aimed at estimating the actual number of steps in chains of acquaintances. In an article published in the popular newsstand magazine *Psychology Today*, Milgram presented the methodological outlines and results of "the first empirically-created chains between persons chosen at random from a major national population"(Milgram, 1967, p. 67).

> Let us assume for the moment that the actual process of establishing the

> linkages between two persons runs only one way: from person A to person Z. Let us call person A the *starting* person, since he will initiate the process and person Z the *target* person, since he is the person to be reached. (...) The general idea was to obtain a sample of men and women from all walks of life. Each of these persons would be given the name and address of the same target person, a person chosen at random who lives somewhere in the United States. Each of the participants would be asked to move a message toward the target person, using only a chain of friends and acquaintances. Each person would be asked to transmit the message to the friend or acquaintance who he thought would be most likely to know the target person. Messages could move only to persons who knew each other on a first-name basis (Milgram, 1967, p. 63).

In a first experiment, 145 participants living in Wichita, Kansas had to reach the wife of a divinity school student living in Cambridge, Massachusetts. The first chain was completed no more than four days after the folders were sent and implied as little as two intermediaries to get completed: "the document had started with a wheat farmer in Kansas. He had passed it on to an Episcopalian minister in his home town, who sent it to the minister who taught in Cambridge, who gave it to the target person" (Milgram, 1967, p.64-65). In a second experiment, done in collaboration with Jeffrey Travers, 160 participants were told to contact a stockbroker working in Boston and living in Sharon, Massachusetts. In total, 44 chains reached the target; their length varied from 2 to 10 intermediate acquaintances, with a median of 5.

Despite the fact that many experimental details and results were left out of the discussion, this pioneering study however

> showed a) that the technique worked and that chains could be completed between widely separated points (e.g., Nebraska and Boston) and b) that the chains could be characterized by certain recurrent features" (Korte and Milgram, 1970, p. 102).

A more rigorous series of experiments was later undertaken by Milgram and Travers, whose results were published in *Sociometry* (Travers and Milgram, 1969). In these experiments, 296 participants were asked to reach the same stockbroker as in the previous study: 96 participants were residents of Omaha, solicited randomly by mail, 100 were systematically chosen Nebraskan stockholders and 100 were volunteers solicited through an advertisement in a Boston newspaper. Again, participants received by mail a document informing them about the target person's name, occupation and place of residence and instructing them to either mail the document directly to that person if they know him personally, or to send it to a personal acquaintance whom is more likely to know him personally. Of the 217 correspondence chains initiated, 64 reached the target person. Analysis of these completed chains revealed two interesting facts. First, the mean and median number of intermediaries between starters and the target is 5.2 and 5; these results, corroborating those of the first study, seemed to indicate that chains could be expected to be completed in six steps, interpretation which gave birth to the famous expression "six degrees of separation" (Guare, 1990). Also, a quarter (16) of the letters mailed directly to the target and thus completing the chains were handled by a single individual, a clothing merchant established in the target's hometown of Sharon; in fact, almost half (31) of the chains reached their target through only 3 individuals. This decrease in the number of persons involved in the completion of chains, similar to a 'funneling effect', points to the key role played by go-betweens, individuals whose large number of contacts allow them to act as social "hubs".

Given these rather surprising results and their global and scientific impact, extensive small-world experimentation was conducted in the following decades. Some of them measured distance between individuals from different social groups: black and white communities (Korte and Milgram, 1970; Lin et al., 1977), Ashkenazi and Mizrahi Jews (Weimann, 1983) and even women and men (Lin et al., 1977). Some small-world studies also experimented with other communication means and purposes, notably telephone calls (Guiot, 1976; Weimann, 1983), complaints-referrals (Stevenson and Gilly, 1991) or emails (Dodds et al., 2003). Other studies, instead of soliciting residential populations, rather focused on more specific kinds of groups or environments, for example organizations of differing bureaucratization types (Lundberg, 1975), hospitals (Stevenson and Gilly, 1991), high-rise buildings (Bochner, 1976) and universities, from classrooms to dormitories (Bochner et al., 1976; Stevenson et al., 1997; Shotland, 1976). Some studies even used fictitious targets in order to collect data on acquaintance volumes and acquaintance chain structures (Bochner et al., 1976; Bochner, 1976; Bochner and Orr, 1979; Killworth and Bernard, 1978).

1.1.2 SMALL-WORLD NETWORKS: THE WATTS-STROGATZ MODEL

By attesting to the existence of both short paths and go-betweens in different social networks, small-world experiments helped stoke the conceptualization of social groups and the society at large as tightly-knit social fabrics (Milgram, 1967, p. 67), in which everyone is connected to everyone in the most surprising and often counterintuitive of ways. However, all these studies lacked explanatory support, and no formal attempts at giving a structural account of the type of configurations and processes involved in experiences and phenomena falling within the scope of small-worldness were made.

In this respect, the real breakthrough came more than 30 years following Milgram's experiments, in a pioneering article published by Duncan Watts and Steven Strogatz. Through reference to both random graph and lattice theories, the authors define small-world networks as graphs that "can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs" (Watts and Strogatz, 1998, p.440). Characteristic path length, noted L, here refers to "the number of edges in the shortest path between two vertices, averaged over all pairs of vertices" (Watts and Strogatz, 1998, p. 440). As for the average clustering coefficient of a network, C, the authors define it as follows:

> Suppose that a vertex v has k_v neighbours; then, at most $k_v(k_v-1)/2$ edges can exist between them (this occurs when every neighbour of v is connected to every other neighbour of v). Let C_v denote the traction of these allowable edges that actually exist. Define C as the average of C_v over all v (Watts and Strogatz, 1998, p.441).

In order to facilitate understanding of both structural indicators, Watts and Strogatz apply them to the analysis of a friendship network: in such intuitive context,

> L is the average number of friendships in the shortest chain connecting two people; C_v reflects the extent to which friends of v are also friends of each other; and thus C measures the cliquishness of a typical friendship circle (Watts and Strogatz, 1998, p.441).

As regards to both measures and their respective definition, random graphs and lattices are diametrically opposed: in the former, clustering and characteristic path length are both minimal, while the highly-clustered nature of the latter and the absence of any shortcuts therein results in long distances between nodes and thus a rather high characteristic path length.

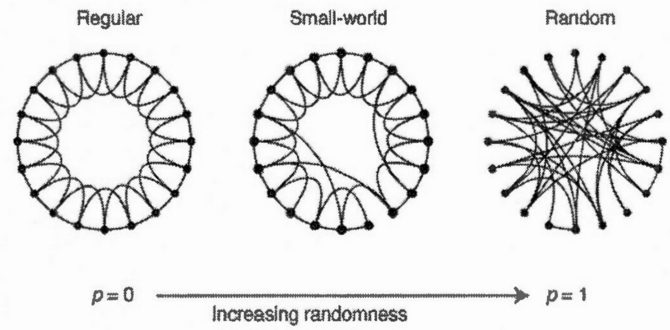
In order to better assess the structural properties of networks lying between these two extremes, the authors devised a network model in which each edge of a regular lattice is randomly rewired with probability p in order "to introduce increasing amounts of disorder" (Watts and Strogatz, 1998, p.440), without however altering the number of vertices or edges. This random rewiring procedure proceeds as follows:

> We start with a ring of n vertices, each connected to its k nearest neighbours by undirected edges (...). We choose a vertex and the edge that connects it to its nearest neighbour in a clockwise sense. With probability p, we reconnect this edge to a vertex chosen uniformly at random over the entire ring, with duplicate edges forbidden; otherwise we leave the edge in place. We repeat this process by moving clockwise around the ring, considering each vertex in turn until one lap is completed. Next, we consider the edges that connect vertices to their second-nearest neighbours clockwise. As before, we randomly rewire each of these edges with probability p, and continue this, process, circulating around the ring and proceeding outward to more distant neighbours after each lap, until each edge in the original lattice has been considered once. (As there are $nk/2$ edges in the entire graph, the rewiring process stops after $k/2$ laps.) (...). For $p = 0$, the original ring is unchanged; as p increases, the graph becomes increasingly disordered until for $p = 1$, all edges are rewired randomly. (Watts and Strogatz, 1998, p.441)

As is the case with regular, lattice-like and random networks, application of this rewiring procedure on networks with $n = 20$, $k = 4$, and rewiring probability values $p = 0$ and $p = 1$ (as shown in Figure 1), results in the generation of networks that present diametrically opposed structural properties. More precisely,

> we find that $L \sim n/2k \gg 1$ and $C \sim 3/4$ as $p \to 0$, while $L \approx L_{random} \sim \ln(n)/\ln(k)$ and $C \approx C_{random} \sim k/n \ll 1$ as $p \to 1$. Thus the regular lattice at $p = 0$ is a highly clustered, large world where L grows linearly with n, whereas the random network at $p = 1$ is a poorly clustered, small world where L grows only algorithmically with n. (Watts and Strogatz, 1998, p.440)

Figure 1 – Random rewiring procedure (Watts and Strogatz, 1998, p.441)



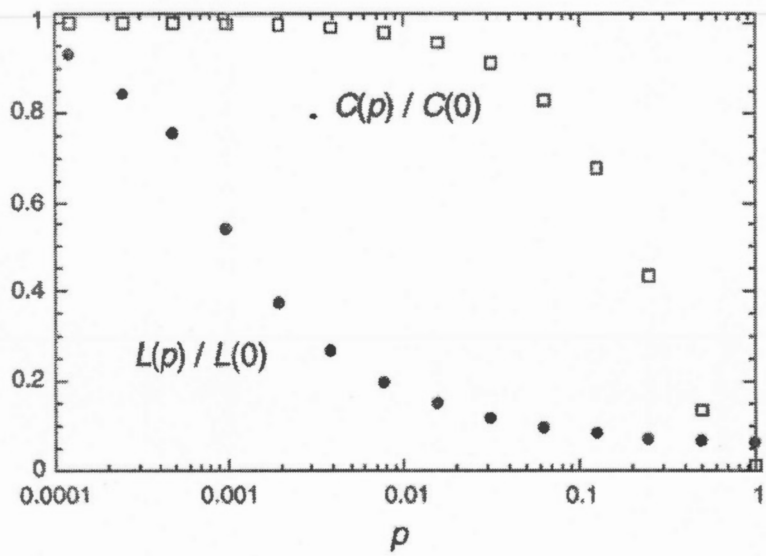
While consideration of these extreme cases seems to suggest that "large C is always associated with large L, and small C with small L" (Watts and Strogatz, 1998, p.440), execution of the same rewiring procedure with probability values $0 < p < 1$ results in the generation of networks with striking and even counterintuitive structural properties, "about which little is known" (Watts and Strogatz, 1998, p.440). Indeed, for a broad interval of p, resulting networks have a characteristic path length comparable to that of a random graph of the same size and order, $L(p) \sim L_{random}$, but having a far more clustered local structure than the latter, $L_{random} \gg C_{random}$. According to the authors, this structural peculiarity results from the introduction of long-range edges.

> Such 'short cuts' connect vertices that would otherwise be much farther apart than L_{random}. For small p, each short cut has a highly nonlinear effect

> on L, contracting the distance not just between the pair of vertices that it connects, but between their immediate neighbourhoods, neighbourhoods of neighbourhoods and so on. By contrast, an edge removed from a clustered neighbourhood to make a short cut has, at most, a linear effect on C; hence, $C(p)$ remains practically unchanged for small p even though $L(p)$ drops rapidly (Watts and Strogatz, 1998, p. 440).

This structural tendency, called "small-world phenomenon" in explicit reference to the social experiments reported in Milgram (1967) and Kochen (1989), is well illustrated in Figure 2, where $C(0)$ and $L(0)$ represent the C and L values of rewired networks with n=1000, $k = 10$, and probability value $p = 0$, averaged over 20 rewiring procedures. In this figure, the use of a logarithmic horizontal scale for p values is testimony to the rapid drop in $L(p)$, characteristic of so-called small-world networks. Most importantly, $C(p)$ remains relatively close to $C(0)$, thus "indicating that the transition to a small world is almost undetectable at the local level" (Watts and Strogatz, 1998, p.441).

Figure 2 – Transition from regularity to disorder in networks (Watts and Strogatz, 1998, p.441)



Thus, the randomly rewired model of Watts and Strogatz hints at the existence of a new kind of network, called 'small-world network' by the authors, which is as clustered and orderly as regular lattices, yet akin to random graphs in the fact that any randomly selected vertex is reachable by a relatively small number of steps. Given that a few short cuts is enough to trigger "small-worldness" in networks, Watts and Strogatz hypothesize that the small-world phenomenon, "is not merely a curiosity of social networks nor an artefact of an idealized model", but might rather prove "generic for many large, sparse networks found in nature" (Watts and Strogatz, 1998, p.441).

In order to validate this claim, the authors compared the average clustering coefficient and characteristic path length of the neural network of the worm *Caenorhabditis elegans*, the power grid of the Western United States, and the collaboration graph of film actors to those of random graphs of equivalent size and order.

> All three graphs are of scientific interest. The graph of film actors is a surrogate for a social network, with the advantage of being much more easily specified. It is also akin to the graph of mathematical collaborations centred, traditionally, on P. Erdős (...). The graph of the power grid is relevant to the efficiency and robustness of power networks. And *C. elegans* is the sole example of a completely mapped neural network.

According to Watts and Strogatz, all these networks had similar characteristic path length and significantly higher clustering score than their random counterparts; given these results, all three were thus considered as being of the small-world kind.

Of particular interest to the present research, both authors then proceed to the description of a series of simulation experiments intended to evaluate the extent to which network dynamics can be reduced to " an explicit function of structure" (Watts and Strogatz, 1998, p.442). More precisely, the "significance of small-world connectivity for dynamical systems" (Watts and Strogatz, 1998, p.441) was addressed in four specific, network-related, cases: disease propagation, density classification, cooperation evolution in prisoner's dilemma on graphs as well as synchronization in coupled oscillatory systems. In general, these simulations have shown that

> "models of dynamical systems with small-world coupling display enhanced signal-propagation speed, computational power, and synchronizability. In

> particular, infectious diseases spread more easily in small-world networks than in regular latices" (Watts and Strogatz, 1998, p.440)

The publication of Watts and Strogatz' article constitutes one of the biggest science buzzes of the new millennium. Between 1999 and 2009, "probably more articles carried out the words "small-world" in their title or abstract than in the 40 years before"(Schnettler, 2009, p. 167). In fact, the scientific effervescence following the publication of Watts and Strogatz's paper is such that many have associated it with the emergence of a "new science of networks" (Barabási, 2002; Watts, 2004; Newman, 2009). Small-world-related structural properties were identified among social networks as diverse as file sharing communities (Iamnitchi, 2011), open source software development communities, networks of scientific collaboration (Newman, 2001a,b) and citation (Bilke and Peterson, 2001; Wallace et al., 2012), corporate board interlocks (Davis et al., 2003), and ownership links among German firms (Kogut and Walker, 2001). But more importantly, thanks to the generality of the Watts-Strogatz model, the small-world effect has gone far beyond the social realm to which it initially and strictly referred to, being discovered in structures and processes as diverse as economic networks (Latora and Marchiori, 2003), Internet and communication networks (Pastor-Satorras et al., 2001; Pastor-Satorras and Vespignani, 2004), epidemic spreading (Pastor-Satorras and Vespignani, 2001; Moore and Newman, 1999), metabolic networks (Jeong et al., 2000; Bilke and Peterson, 2001; Wagner and Fell, 2001), protein interaction networks (Jeong et al., 2001), food webs (Montoya and Solé, 2002) as well as diverse cerebral structures (Humphries et al., 2006; Bassett et al., 206; Achard et al., 2006).

Such effervescence and diversity of interest certainly shows that 'topological structure of many real-world networks are more similar than one would have expected (Lin and Ban, 2013, p.658). But also and more importantly, this whole research program is testimony to the structural efficiency of small-world networks, especially as regards to interaction and movement facilitation. In light of this, it shouldn't come as a surprise that a significant amount of recent research in transport studies is specifically aimed at discovering small-world structures in various transportation systems.

1.1.3 SMALL-WORLDS IN TRANSPORTATION SYSTEMS

Emblematic of spatial networks, transportation refers to the process of conveying "energy, matter or information from one point to another"(Barthélémy, 2011, p.13). While this broad definition allows many of the above cases to qualify as transportation, the latter term is often limited to cases implying movement of matter. But even with this stricter definition, transportation systems still refer to a great variety of structures, for example city streets (Buhl et al., 2006; Cardillo et al., 2006), plant leaves (Runions et al., 2005), river networks (Rodríguez-Iturbe and Rinaldo, 2001), mammalian circulatory systems (West and Brown, 2005), networks for commodities delivery (Gastner and Newman, 2006), technological networks (Schwartz, 1987), and so on.

Given the importance of movement and translocation for transportation networks such as these, cost-effectiveness and navigability constitute key imperatives. Take the human brain for example. Containing about 10^{10} neurons and 10^{14} connections, the human brain is one of the most complex networks known. Here as elsewhere, space is of the essence, as "mobility trades space for a cost"(Rodrigue et al., 2009, p. viii).

> Consider a toy model of the brain. Let us assume that is consists of local processing units, connected by wires. What constraints act on this system? On the one hand, one would want the highest connectivity between the local processing units so that information could be exchanged as fast as possible. On the other, it is wasteful to wire everything to everything else. The energy requirements are higher, more heat is generated, and more material needs to be used, and consequently, more space is occupied (Mathias and Gopal, 2001, p.2)

In spatially embedded systems such as this one, cost-effectiveness and navigability constitute powerful constraints on structure efficiency: first, close brain regions have a larger probability of connection than remote ones, as longer axons are more costly in terms of material and energy (Bullmore and Sporns, 2009); but also, since long connections are expensive, longer links must be compensated by some advantage, such as for example being connected to a highly-connected node - that is, a hub (Barthélémy, 2011). Given that structural analyses of brain networks indicate that these structures have small world properties, being both characterized by a large clustering coefficient and a small characteristic path length (Eguiluz et al., 2005; Kaiser and Hilgetag, 2004), small world networks must confer a comparative advantage in terms of transportation over other types of reticular structures. Many scholars have pointed out that, from a neural network point of view, small worlds represents an ideal balance between local processing (clusters) and global integration (shortcuts and hubs), ensuring simultaneously rapid synchronization, information transfer and resilience to damage. (Barthélémy, 2011; Lago-Fernández et al., 2000). In short, from a neural perspective, small world networks constitute optimal transportation infrastructures.

As with brain networks, every transportation network is subjected to the same dual constraint of connectivity maximization and cost minimization. On the one hand, shortest paths play an important role in the transport and communication within a network: "Suppose one needs to send, e.g., a data packet from one computer to the other through the Internet. The shortest path provides an optimal path way since one would achieve a fastest transfer and save system resources" (Noh and Rieger, 2002, p. 1). On the other hand, all transportation networks are subjected to distance costs, which impose strong limits on the number of possible connections. Given that these constraints are the same as those observed for neural networks, there is reason to think that the relative fitness of small-world networks in terms of transportation efficiency also applies to transportation systems in general: by having a large number of local connections and a few long-range links connecting local clusters together, small-world networks have been shown to be efficient at both the local and global level, thus making them optimal structures in terms of navigation (Kleinberg, 2000; Gorman and Kulkarni, 2004; Csanyi and Szendroi, 2004).

Such reasoning might explain why, in geography of transportation and transportation research, where the imperatives of navigability are obviously crucial, intense scholarly effort has been expended at uncovering small-world-like patterns in various ground, air, and maritime networks (Lin and Ban, 2013). As regards to railway systems, small-world properties were detected in the Indian (Sen et al., 2003) and Chinese (Li and Cai, 2007; Wang et al., 2008) networks, as well as in the railway network of metropolitan Tokyo (Majima et al., 2007). High average clustering and low characteristic path lengths, indicative of small world structures, have also been observed for the worldwide air transportation network (Barrat et al., 2004; Guimera and Amaral, 2004; Guimera et al., 2005) and for the internal networks of Italy (Guida and Maria, 2007), India (Bagler, 2008), China (Li and Cai, 2004; Lin, 2012; Wang et al., 2011) and America (Paleari et al., 2010). Given these results, small-world networks have been acknowledged as efficient for aviation networks (Chi et al., 2003; Hong-Kun and Tao, 2007). As regards to maritime transportation networks, which carry as much as 90% of the world's trade (Kaluza et al., 2010), analysis of both worldwide (Hu and Zhu, 2009) container liner transportation and Chinese ship transport networks (Xu et al., 2007) have also shown small-world properties.

In the particular case of urban road networks, research has led to widely diverging results, depending on the modeling procedure used: traditional, metric-based road networks do not present small-world-like properties, while street graphs, based on user-centered and street-based representations of road networks, present totally opposite results in this regard. The present research aims at providing in-depth knowledge of such contrast. A detailed description of both modeling procedures is given in the following section.

1.1.4 ROAD-BASED AND STREET-BASED MODELS OF URBAN ROAD NETWORKS

Contemporary urban studies literature provides two main approaches to graph-based representation of road networks: a geometrical one, based on Euclidean geometry and metric invariance, and a topological one, focusing on relational patterns between streets. While the first represents the most ancient and popular one, the second has been shown to correspond more adequately to the experiential and user-centered dimension of urban networks.

1.1.4.1 ROAD NETWORKS

The link between road systems analysis and network theory has a rather long history. In 1736, Leonhard Euler published his proof of the unsolvability of the Königsberg's bridges problem, which consisted in finding a round trip that traversed each of the seven bridges of this city exactly once. To solve this problem, the Swiss mathematician converted locations into points, and bridges between locations into links connecting the corresponding nodes, and such that no two curves meet in a point other than a common end. Such formalization procedure resulted in the creation of a mathematical structure of utmost geometrical abstraction, focusing solely on adjacency relations between entities: a graph.

With this mathematical initiative, Euler inaugurated both graph theory and transportation network analysis at the same time. While this co-creative act certainly contributed to make their joint use look intuitive, even natural, it also had a lasting impact as regards to formalization and modeling processes. In order to represent as adequately as possible real-world road systems, transportation research has indeed exclusively relied on the same roads-as-links approach as the one used by Euler (Kansky, 1963; Chorley and Haggett, 1967; Haggett and Chorley, 1969).

> All graph edges are defined by two nodes (the endpoints of the arc) and, possibly, several vertices (intermediate points of linear discontinuity); intersections among edges are always located at nodes; edges follow the footprint or real streets as they appear on the source map; all distances are calculated metrically (Porta et al., 2006b, p.712).

This retainment of geometric patterns and geographical properties, resulting in the creation of so-called "geometric graphs" (Courtat et al., 2011a; Barthélémy, 2011), rapidly became a benchmark in network implementation of transportation and geographical systems (Crucitti et al., 2006b; Porta et al., 2006b; Scellato et al., 2006) as well as "the world standard in geospatial dataset construction and diffusion" (Porta et al., 2006b, p. 711).

The geographical embedding of geometric models however strongly constrains their topological structure. On the one hand, the number of edges that can be connected to a single node is limited by the physical space to connect them (Barthélémy, 2011; Boccaletti et al., 2006). As regards to road networks in particular, analysis of 20 German cities has shown that most nodes have four neighbours, a number of connections that rarely exceeds 5 for various other world cities (Lämmer et al., 2006). Also, two distant nodes are less likely to be connected due to the distance dependent cost of the edges (Boccaletti et al., 2006). Thus, both the number of edges and long-range connections that can be connected to a single node are limited by the geographic embedding of geometric graphs, thus excluding the possibility of small-worldness altogether (Barthélémy, 2011; Boccaletti et al., 2006; Cardillo et al., 2006; Csanyi and Szendroi, 2004; Crucitti et al., 2006a). Given the omnipresence and proven navigability of small-world networks, the idea that transportation infrastructures don't present small-world properties seems counter-intuitive, to say the least.

1.1.4.2 STREET NETWORKS

Studies have often pointed out that geometric models of road networks weakly represent the way people experience such networks (Jiang, 2013). As one author aptly pointed out, they violate "the intuitive notion that an intersection is where two roads cross, not where four roads begin" (Kalapala et al., 2006, p.1). Two different but related ideas seem to be implied by this remark: (1) roads constitute the fundamental components of road networks, intersections being only the emerging result of their entanglement and crisscrossing; (2) road identity is not delimited by intersections but often extend over multiple segments.

The graph-theoretical implementation of these two ideas resulted in the development of so-called street-based models [1]. As first suggested by Jiang and Claramunt (2002),

1. In transportation network analysis, this new urban street network modeling technique has often been referred to as the "dual approach". From a graph-theoretical point of view, such an expression is however misleading, as the "dual" of a plane graph is a graph having respectively for vertices and edges

treating roads as fundamental topological entities suggests a new kind of network representation, in which nodes stand for the road themselves, and two nodes are connected if there exists an intersection between the two corresponding roads. As for the second idea, it implies that, in order to make better sense of road networks, individual segments need to be merged into meaningful ordered sets: streets.

Defined as such, streets are closely related to cartographic generalization of linear objets such as hydrological networks: in order to reduce cartographic complexity in scale-reduction processes and thus facilitate global map legibility, such generalization proceeds by merging segments into longer "strokes", which are then sorted hierarchically by structural importance (Porta et al., 2006a). Here as in street-based models, generalization is closely tied to cognitive factors; however, in the latter case, the main objective is not to allow for better visualization, but to build representations that better reflect the experience of road networks by its users. (Porta et al., 2006a; Batty and Rana, 2004; Jiang and Claramunt, 2002; Jiang, 2007, 2013; Penn, 2003).

In order to "recover the actual streets of a city" (Courtat et al., 2011c, p.3), two generalization models have been proposed in the literature, each of them representing a distinct way of defining "which road segments naturally belong together" (Kalapala et al., 2006, p.2). The first generalization model developed for street topologies is based on street names: two different road segments are assigned the same street identity if they share the same odonym or street name (Jiang and Claramunt, 2004b,a; Kalapala et al., 2006). Through their names, streets become meaningful and urban networks are given a semantic dimension. Odonyms being often the only street property that is part of common knowledge, they can easily be used for communicative purposes (Tomko et al., 2008, p.44).

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the faces of that plane graph. The term "dual" is used in this case because the relationship is symmetric: if one graph is the dual of another, the latter is also its dual. Street-based networks are certainly not dual in this sense; in fact, a proper dual of a plane graph would have one node for each face of the planar graph and one link between two nodes if the faces to which they correspond are adjacent. In the case of real-world urban road network, faces would stand for city blocks and thus correspond to urban features such as buildings, houses and parks.

> When giving directions we do not describe every intersection, and do not account for every street segment through which we pass. Rather, we typically instruct people to follow linear axes, and single out only those intersections where one should make a turn, that is, cross from one axis to another. Intersections that connect two segments of the same axis or street are usually ignored. In terms of way-finding and urban orientation, then, urban linear axes can be viewed as constitutive units, which are related to each other by means of intersections (Wagner, 2008, p.2121)

A second generalization model was later developed, based on angular information (Porta et al., 2006a): adjacent segments meeting a specific condition relative to their incidence angle may be merged together as part of the same angular street. Angularity-based streets, also referred to in the literature as 'continuity lines' (Figueiredo and Amorim, 2005), are based on a well-known cognitive principle of human wayfinding, which is the tendency to go straight at intersections (Conroy Dalton, 2003; Dalton, 2001; Dalton et al., 2003). In Turner (2009), path analysis of 2425 individual motorcycle trips made in London by motorcycle couriers has indeed revealed that as much as 63% of them took the minimum possible angular distance between origin and destination, while only 51% of trips followed the minimum possible block distance.

Regardless of the generalization model, be it nominal or angular, street networks present structural properties that are radically different from those of metric-based road networks. Distance loses its metric meaning and becomes equivalent to the number of 'steps' separating one street from another street, regardless of length of those steps (Porta et al., 2006b)[2]. In this sense, street topologies offer " an information view of the city, where distances along each road are effectively set to zero because it does not demand any information handling to drive between the crossroads"(Rosvall et al., 2005, p.1). Also, by allowing the identity of streets to span a theoretically unlimited number of intersections, odonymic or angular generalization allows corresponding nodes in the street network to be incident to a theoretically unlimited number of edges.

2. However, metric information can still be derived from topological distance, given that line length can roughly correlate with the number of connections of the corresponding node (Wagner, 2008).

This loss of geometric constraints regarding both distance and node degree, by making street networks structurally similar to complex systems not subjected to constraints related to geographical space (Porta et al., 2006b,a), has led to significant results regarding road network small-worldness. In Jiang and Claramunt (2004b), the named-based topology of the street networks of Gävle, Munich and San Francisco have been shown to reveal small-world properties. It has also been shown in (Porta et al., 2006a) that the angular topology of 1-square mile samples taken from the street networks of Ahmedabad, Venice, Vienna and Walnut Creek (CA) possess small-world properties. More recently, both odonymic networks and angular street networks with threshold of 60^o of the city of Hong Kong have shown small-world properties (Jiang and Liu, 2009).

In part due to these impressive results, street-based modeling have been said "to have sustained the by far most relevant, if not the sole, specific contribution of urban design to the study of city networks" (Porta et al., 2006a). However, both the significance and scope of these results are hampered by the fact that street-based topological modeling conventionally relies on undirected graphs (Penn, 2003), which reduces accessibility between streets to symmetrical relationships and thus leave out a crucial aspect of urban network: the asymmetry degree of road networks (Jiang, personal communication). Such overlooking should certainly come as a surprise, given the asymmetry degree of urban networks is usually far from negligible: some road sections, aptly named 'two-ways', allow vehicles to circulate in both ways while others, called 'one-ways', are strictly unidirectional. This non-symmetry of urban road networks due to road direction represents the main focus of the present research.

1.1.5 ROAD DIRECTION AND THE NON-SYMMETRY OF ROAD NETWORKS

Asymmetric relations represent a key aspect of many structural and dynamical phenomena and processes:

> It is well-known that many real-world complex networks involve non-mutual relationships, which imply non-symmetric adjacency or weight matrices. For instance, trade volumes between countries are implicitly directional relations, as the export from country i to country j is typically different from the export from country j to country i (i.e. imports of i from j). If such networks are symmetrized (e.g., by averaging imports and exports of country i), one could possibly underestimate important aspects of their network architecture (Fagiolo, 2007, p. 3).

Asymmetric relations such as those are rather prevalent in real-world networks, and taking them into account has become a necessity in many other areas of network research, as demonstrated by the frequent use of directed or mixed graphs. In the case of road networks, graph theory has shown an early interest in one ways, as it exemplifies and constitute an immediate real-world application of graph orientation problems. Robbin's aptly-named "one-way theorem" (Koh and Tay, 2002), according to which "a graph is orientable if and only if it remains connected after the removal of any arc" (Robbins, 1939, p.281), represents a well-known case in this regard.

> Let us suppose that week-day traffic in our city is not particularly heavy, so that all streets are two-way, but that we wish to be able to repair any one street at a time and still detour traffic around it so that any point in the city may be reached from any other point. On week-ends no repairing is done, so that all streets are available, but due to the heavy traffic (perhaps it is a college town with a noted football team) we wish to make all streets one-way and still be able to get from any point to any other without violating the law. Then the theorem states that if our street-system is suitable for week-day traffic it is also suitable for week-end traffic and conversely (Robbins, 1939, p.281-282).

Boesch and Tindell (1980) extended Robbin's research to mixed multigraphs, while Chung et al. (1985) provided a linear-time algorithm to detect whether a mixed graph has a strong orientation or not and finding one if it does. Robbin's research also led to fruitful investigations regarding optimal orientation of graphs, that is, strong orientations of graphs having minimal longest distance between two vertices [3]. On this matter, Chvátal and Thomassen (1978) have proven that the problem of deciding whether a graph admits an orientation with longest distance of only two is NP-hard.

Closer to the present research, a series of articles by Roberts and Xu (Roberts and Xu, 1988, 1989, 1992, 1994) investigated further the problem of converting a two-way road system into a one-way network and analyzed the effect of this orientation on the distance between locations within the network. Traditional one-way layout in which parallel ways alternate in direction is not optimal in keeping the distance as short as possible. In fact, by optimizing the strong orientation of graphs through the minimization of the longest distance between two vertices, the authors found out that the best orientation included intersections where one-ways meet head-on.

However and despite these various mathematical examples, the neglect of road asymmetry is far from uncommon. For example, road transportation investigations "do not actually study (...) the effect that different degrees of asymmetry (...) have over solution methodologies" (Rodríguez and Ruiz, 2012a, p.1567). In this regard, two studies need to be mentioned here. The first study investigates the impact of road asymmetry on the traveling salesman problem. The Traveling Salesman Problem consists formally in finding the shortest Hamiltonian cycle for a given network, that is, the shortest cycle that visit each node of the network exactly once:

> "Consider a salesperson who wants to travel around the country from city to city to sell his wares. (...) The traveling salesperson does not want to visit any city twice and at the end of his trip he wants to return to the same city he started in. The question is what route can the salesperson take to exhaustively visit all the cities without going through the same city twice. It is also in the salesperson's best interest to spend the least amount of time traveling, therefore, he would like to cover the least possible total distance. In order to facilitate this need, distance or travel time should be incorporated in the map as edge cost. More formally, the problem can be

3. As will be seen in section 3.4, this corresponds the lowest possible diameter of a graph

> considered as a connected graph where the nodes are the cities and the edges are the roads between them. Each edge has a weight associated with it that is analogues to a distance. The goal of the problem is to visit all of the nodes without visiting any twice and do this while traveling the dyes but incurring the minimum cost" (Wilfahrt and Kim, 2008, p. 1173-1174).

As for the second study, it focuses on the impact of road asymmetry on real capacitated routing problems:

> "The Vehicle Routing Problem (VRP) embraces a class of complex combinatorial optimization problems that target the derivation of mimimum total cost routes for a number of resources (vehicles) located at a central point (depot) in order to service efficiently a number of demand points (customers). The standard version of VRP (known as basic VRP) is defined on a graph $G = (V,A)$, where $V = \{u_0, u_1, ..., u_n\}$ is the vertex set and $A = (u_i, u_j) : u_i, u_j \in V, i \neq j$ is the arc set of G. Vertex u_0 represents a depot (warehouse or distribution centre) that hosts a homogeneous fleet of m vehicles with capacity Q. The remaining vertices correspond to demand points (or equivalently, customers). Each customer u_i has a non-negative demand q_i. The vector of all customer demands is denoted by $q(V)$. Furthermore, a non-negative cost matrix $C = (c_{ij})$ is defined on A; usually, the cost c_{ij} models the travel time between customers u_i and u_j. If $c_{ij} = c_{ji}$, the problem is symmetric, and it is common to replace A with the edge set $E = (u_i, u_j) : u_i, u_j \in V, i \not< j$. The solution to the basic VRP is a set of routes that satisfy the following constraints: a) each route starts and ends at the central depot; b) each customer is visited exactly once; c) every customer's demand is satisfied; d) the total travel time of the set of routes is minimized" (Tarantilis, 2008, p. 979). In the case of the Capacitated Vehicle Routing Problem (CVRP), an additional constraint is imposed: "the total demand of the customers covered by a route cannot exceed the capacity of a vehicle Q" (Tarantilis, 2008, p. 979).

In both cases, results have shown that road asymmetry degree greatly impacts on algorithm performance and efficiency as well as solution quality (Rodríguez and Ruiz, 2012a,b). Given this, the overlooking of road direction might well prove to be a significant shortcoming of street-based modeling of road networks, particularly as regards to the evaluation of the small-worldness of street graphs.

In order to properly evaluate the impact of road asymmetry on street network small-worldness, however, accurate and precise quantitative methods are needed. In fact, the Watts-Strogatz model on which most small-world studies are based can only vaguely determine small-worldness: given that only a few random ties in an otherwise ordered network are needed for low characteristic path lengths to emerge, "a surprisingly low threshold condition for the existence of the small-world effect" (Schnettler, 2009, p. 169), any graph that is structurally midways between disordered random graphs and ordered lattices qualify as a small-world network. Thus, in order to evaluate both the impact of road direction on street graphs as well as the small-worldness of real-world networks, a proper small-worldness measure is needed. To address both needs, a new structural indicator, initially designed to measure the small-worldness of brain networks, can be of great use: the ω metric.

1.1.6 FROM SMALL-WORLD NETWORK TO NETWORK SMALL-WORLDNESS: THE ω METRIC

While the definition of 'small-worlds' given in Watts and Strogatz (1998) allows for a more complete understanding of networks, by breaking "the continuum of network topologies into the three broad classes of lattice, random graphs and small-worlds, the latter being by far the broadest" (Humphries and Gurney, 2008, p. 1), the definition of small-world networks is still rather imprecise: while both average clustering coefficient and characteristic path length respectively give a good impression of the local and global orderliness of networks, the definition given by Watts and Strogatz (1998) "leads to a categorical distinction ('small/not-small') rather than a quantitative, continuous grading of networks, and can lead to uncertainty about a network's small-world status" (Humphries and Gurney, 2008, p.1).

In addition to these definitional shortcomings, procedural issues pertaining to small-world identification also needs to be stressed. While in principle, small-world networks have lattice-like average clustering coefficients, in practice, lattices are not even considered in analyzing network structures, as "networks are typically defined as small-world by comparing clustering and path length to those of a comparable random network" (Telesford et al., 2011, p.367). Case studies made by Watts and Strogatz themselves are no exception to this trend: while referring to lattice clustering in the definition, both authors nevertheless revert to the average clustering coefficient of random graphs in their empirical analyses of films actor, power grid and *C. elegans* neural networks (Watts and Strogatz, 1998). This inconsistency between words and actions certainly appears as surprising:

> "One possible reason why comparisons with network lattices have not been used in the the literature up to this point is the length of time it takes to generate lattice networks, particularly for large networks. One appeal of comparing the original network to only a random network is rather fast processing time to generate the random network"(Telesford et al., 2011, p.373).

Regardless of the reason, however, such practice proves detrimental to rigorous network analysis, since "the comparison of clustering to an equivalent random network does not properly capture small-world behavior because clustering in a small-world network more closely mimics that of a lattice network" (Telesford et al., 2011, p.368).

In order to cope with both definitional and procedural issues, a new small-worldness quantitative measure, called the ω metric, has been designed by Telesford et al. (2011). Given a graph with characteristic path length L and average clustering coefficient C, the small-worldness metric ω is defined by comparing C to the average clustering coefficient of an "isosequential" (i.e., with same number of nodes and links as well as identical degree sequence) highly-clustered, lattice-like, graph, C_{latt}, and L to the characteristic path length of an 'isosequential' random graph, L_{rand}. The value of ω is the difference between the two ratios thus established:

(1) $$\omega = \frac{L_{rand}}{L} - \frac{C}{C_{latt}}$$

Values of ω are restricted to the interval $[-1,1]$. The supremum and infimum correspond respectively to optimal, lattice-like, orderliness and random-like disorderliness for a given degree sequence. In this sense, while the Watts-Strogatz model has helped place networks "along a continuum from lattice to small-world to random"(Telesford et al., 2011, p. 368), the ω metric simply puts that continuum into a quantitative framework. Positive ω scores indicate disorderly and random network structures, satisfying $L \gtrsim L_{rand}$ and $C \ll C_{latt}$, while negative ω scores are indicative of ordered, regular and lattice-like graphs, with $L \gg L_{rand}$ and $C \lesssim C_{latt}$. Also, in accordance with the definition given in Watts and Strogatz (1998), the closer an ω score is to zero, the more $L \gtrsim L_{rand}$ and $C \lesssim C_{latt}$ hold and the higher the small-worldness of the network analyzed. And in case a network's ω score approximates the zero point, its structure is balanced to the point that "characteristic path length is as close to random as clustering is to a lattice"(Telesford et al., 2011, p. 370). As regards to determining whether a given network is a small-world or not, no precise cut-off points are given by the authors; "however, should one decide that a cut-off is desirable, the small-world region approximately spans the range $-0.5 \leq \omega \leq 0.5$" (Telesford et al., 2011, p.370).

The ω metric does have some drawbacks, especially in the case of small networks or networks with extremely low average clustering coefficient:

> Observations of ω show that as network size decreases, the range of ω tends to decrease. (...) This occurs because in smaller networks, the equivalent lattice tends to have shorter characteristic path length. Because the path length in the lattice network is closer to that of the random network, L_{rand}/L (...) deviates from 0; thus, ω does not approach -1. ω is also limited by network that have very low clustering that cannot be appreciably increased, such as networks with "super-hubs" or hierarchical networks (Telesford et al., 2011, p.373).

These shortcomings have no impact on the present objectives, however. As chapter IV will show, all graphs generated and analyzed in this research contain thousands of nodes and vertices and their average clustering coefficient is high enough to allow for the generation of isosequential highly-clustered graphs. As for the absence of any clear-cut small-worldness boundaries, this in no way implies that the ω metric can't be used for efficient comparative analysis. On the contrary, small-worldness can be duly evaluated by referring to both the ω supremum and infimum as well as the ω scores of other networks.

Overall, the ω small-worldness indicator enhances the applicability of the Watts-Strogatz model by offering a precise and accurate means to quantify, evaluate and compare network small-worldness. As the creators of the ω metric themselves show, by applying the ω metric to many real-world networks, the authors show that "small-world networks are not as ubiquitous as reported", which "suggests that many systems originally thought to have small-world processing capabilities may in fact not" (Telesford et al., 2011, p. 268). Since no such analyses were done in the case of urban networks and topological graphs in particular, the task of proving their true small-worldness still remains to be undertaken. In light of this, assessment of the structural impact of road asymmetry on street-based models based the use of the ω metric becomes an even more relevant and pressing task.

CHAPTER II

METHODOLOGICAL PRELIMINARIES: FORMAL DEFINITIONS AND CONCEPTS

In order to avoid any ambiguity or confusion, the present chapter aims to define and justify the essential formal concepts used in the course of this research. As a first step, all logical, set-theoretic and graph-theoretic concepts and relations used for the description and explanation of the different graph generation procedures implemented in this research are presented. Then, given the highly technical nature of the geographical data used in the present research and in order to facilitate the understanding of the data collection, preprocessing and vector map conversion stages, this chapter concludes with a short introduction to both geographic data modeling and vector-based representation of road networks.

2.1 LOGICAL PRELIMINARIES

As the formal systematic study of the principles of valid inference and sound reasoning, "logic has the important function of saying what follows from what" (Kleene, 1967, p.3). Given the deductive nature of mathematics, it shouldn't come as a surprise that logic intervenes ubiquitously in the definition of concepts and operations used in most branches of mathematics as well as knowledge in general.

> Every development of mathematics makes use of logic. A familiar example is the presentation of geometry in Euclid's "Elements" (c. 330-320 B.C.), in which theorems are deduced by logic from axioms (or postulates). But any orderly arrangement of the content of mathematics would exhibit logical connections. Similarly, logic is used in organizing scientific knowledge; and as a tool of reasoning and argumentation in daily life (Kleene, 1967, p.3).

Providing an exhaustive portrait of mathematical logic lies of course beyond the scope of the present thesis. However, given that propositional connectives are amply used throughout the formalisms presented in the following pages, a short digression seemed in order.

The focus here will be on propositional calculus, the branch of logic that studies the truth-functional implications of sentence combination, "in which the truth or falsity of the new sentence is determined by the truth or falsity of its component sentences" (Mendelson, 1979, p.11). To make things short, propositional connectives are here presented by specifying their truth-preserving conditions, that is, the alethic configurations that allow propositions built out from more basic ones to stay true, given the truth value of their components. Also, in order to avoid any confusion with set-theoretic symbols, the present research relies mainly on the logical symbolism used in Gabbay and Günthner (1984). Given these considerations, the notational and semantic conventions shown in table 1 prevail throughout this research.

Indeed, these definitions do not cover all possible alethic 2-permutations; however, given that all set- and graph-theoretic formulas used in the study can be expressed through this restricted set of connectives, their sole presentation suffices for the present needs.

Table 1 – Logical primitives used in this research

Symbol	Connective	Truth-preserving condition
$\neg p$	Negation	Holds if p is false
$p \wedge q$	Conjunction	Holds only if p and q are both true
$p \vee q$	Disjunction	Holds unless both p and q are false
$p \rightarrow q$	Implication	Holds unless p is true and q is false
$p \leftrightarrow q$	Equivalence	Holds if p and q are both true or both false
Symbol	**Quantifier**	**Truth-preserving condition**
$\forall x(p)$	Universal	p holds for all x
$\exists x(p)$	Existential	p holds for at least one x

2.2 SET-THEORETIC PRELIMINARIES

Built on "the ability to regard any collection of objects as a single entity" (Devlin, 1993, p.1), set theory aims at describing the structure of the whole mathematical universe (Jech, 2011, p.1) through the definition of such collections, called sets, and the specification of relations and operations that may apply to them. Very general in scope, set theory represents an ideal tool for the description of most mathematical objects and operations.

> With a few rare exceptions the entities which are studied and analyzed in mathematics may be regarded as certain particular sets or classes of objects. This means that the various branches of mathematics may be formally defined within set theory. As a consequence, many fundamental questions about the nature of mathematics may be reduced to questions about set theory (Suppes, 1960, p.1)

In light of this universal scope, it shouldn't come as a surprise that the whole field of graph theory rests on set-theoretic concepts and definitions. Behind this formal consensus however lies an impressive diversity of notations, definitions and presentation types, from naive introductions based on natural language to sophisticated axiomatizations. Given this, most of the set-theoretic definitions used in this research, mainly derived from Suppes (1960), Jech (2011) and Devlin (1993), are presented in table 2.

Table 2 – Set-Theoretic Concepts and Definitions

Symbol	**Relation**	**Definition**
$\in$	Membership	Given an object x and a set s, if x is an element (or member) of s, then $x \in s$; otherwise, $x \notin s$
$s_1 \subseteq s_2$	Inclusion	$\forall x(x \in s_1 \rightarrow x \in s_2)$
$s_1 = s_2$	Identity	$\forall x(x \in s_1 \leftrightarrow x \in s_2)$
$s_1 \cup s_2$	Union	$\{x \mid x \in s_1 \vee x \in s_2\}$
$s_1 \cap s_2$	Intersection	$\{x \mid x \in s_1 \wedge x \in s_2\}$
$\bigcup s_1$	Set Union	$\{x \mid \exists s_2(x \in s_2 \wedge s_2 \in s_1)\}$
$\bigcap s_1$	Set Intersection	$\{x \mid \forall s_2(s_2 \in s_1 \rightarrow x \in s_2)\}$
$\{x,y\}$	Unordered Pair	$\exists s \forall z(z \in s \leftrightarrow (z = x \vee x = y))$
$\langle x,y \rangle$	Ordered Pair	$\{x, \{x,y\}\}$
$\langle x,y,z \rangle$	Ordered Triple	$\langle \langle x,y \rangle, z \rangle$
$\mathscr{P}(x)$	Set Partition	$\{p \mid (\cup p = x) \wedge$ $\forall y,z((y \in p \wedge z \in p \wedge y \neq z) \rightarrow y \cap z = \emptyset) \wedge$ $\forall a(a \in p \rightarrow \exists b(b \in a))\}$

2.3 GRAPH-THEORETIC PRELIMINARIES

Graphs are mathematical structures used to model pairwise relations or "connections" between objects and analyze the overall topological patterns that result from these pairings. Nowadays one of the most effervescent fields of discrete and applied mathematics, graph theory has become an important formal tool for computer science as well as the foundation stone of "the new science of networks" (Barabási, 2002; Buchanan, 2002; Watts, 2003), whose aim is to investigate interaction patterns in large, complex systems. In spite of, or perhaps even because of this growing interest, no standard terminological and definitional consensus currently exists in this field, as "most graph theorists use personalized terminology in their books, papers, and lectures" (Harary, 1969, p.8).

In the case of mixed graphs in particular, the situation is even more delicate, as such graphs "are known but have not received much attention" (Stiege, 2007, p.3). To our knowledge, the only textbook aimed at offering a substantive formal treatment of mixed graphs is rather recent and hasn't yet been translated from German (Stiege, 2006). Fortunately, the author has written a report "written to give a reference source to readers who do not speak German" (Stiege, 2007, p.4). While this report, as well as major references such as Diestel (2000), Bang-Jensen and Gutin (2010), Chartrand and Lesniak (2000) have been thoroughly consulted, a certain definitional liberty has been taken in this research in order to formalize graph-theoretic concepts in a way that most fits the present purposes.

2.3.1 GEOMETRIC GRAPHS

All graphs generated and analyzed in the present research belong to the group of geometric graphs, "a large, amorphous body of research related to graphs defined by geometric means" (Pach, 2013, p.1). According to Jànos Pach, a graph G is geometric if it is "drawn in the *plane* with possibly intersecting straight-line edges" (Pach, 2013, p.1). Based on this definition, any graph G considered in this research is an element of $\mathbb{G}$, the set of all graphs satisfying the properties given in equation 1.

$$(1) \qquad G \in \mathbb{G} = \left\{ \langle V,A,E \rangle \;\middle|\; \begin{array}{l} V \neq \emptyset \\ (A \cup E) \neq \emptyset \\ V \subseteq \mathbb{R}^2 \\ A \subseteq V^2 \\ E \subseteq [V]^2 \\ (V \cap A) \cup (V \cap E) \cup (A \cap E) = \emptyset \end{array} \right\}$$

Thus, as an element of $\mathbb{G}$, any graph G is an ordered triple of finite sets. For any graph G, sets V,A and E are respectively called the vertex, arc and edge sets of G and referred to as $V(G)$, $A(G)$, and $E(G)$ respectively. Elements $v \in V(G)$ are the vertices of G and referred to as $V(G) = \{v_1, v_2, ..., v_m\}$. Elements $a \in A(G)$ are the arcs of G, $A(G) = \{a_1, a_2, ..., a_n\}$, while elements $e \in E(G)$ are called the edges of G, such that $E(G) = \{e_1, e_2, ..., e_q\}$. The first and second membership criteria of equation 1, $V \neq \emptyset$ and $(A \cup E) \neq \emptyset$, respectively exclude the possibility of empty (without vertices) and trivial (without arcs and edges) graphs, which are not considered in this research and whose further mention might otherwise be confusing. The third membership condition, $V \subseteq \mathbb{R}^2$, states that every vertex $v \in V(G)$ is a pair of coordinates, each one corresponding to a real number. This condition allows for all vertices of all graphs in this research to be embedded as points in the Euclidean plane. Given that all real-world entities considered here constitute geographic entities with precise spatial coordinates, such embedding allows for the graph representation of road networks to be as metrically faithful as possible. As for the fourth membership criterium, $A \subseteq V^2$, it states that the arc set $A(G)$ is a subset of the cartesian square $V(G) \times V(G)$, which corresponds to the set of all ordered pairs (or 2-permutations) of vertices in $V(G)$; as such, all members of $A(G)$ are necessarily ordered pairs, here represented between angle brackets, such as in $\langle x, y \rangle$. Parallel to this, the fifth membership criterium, $E \subseteq [V]^2$, asserts that edge set $E(G)$ is a subset of $[V(G)]^2$, which designates the set of all possible unordered pairs (or 2-combinations) of vertices in $V(G)$. Finally, the sixth membership criterium, $(V \cap A) \cup (V \cap E) \cup (A \cap E) = \emptyset$, says that the vertex, arc and edge sets of any given graph are necessarily and mutually disjoint, which means that any member of one of these sets, be it a vertex, an arc or an edge, is an exclusive member of that set.

Given the conditions specified in equation 1, different types of graphs can be distinguished, depending on the cardinality (zero or non-zero) of both arc and edge sets: an undirected graph contains edges but no arcs, a directed graph contains arcs but no edges, while a mixed graph includes both arcs and edges. As shown in table 3, all three different types of graphs will be used in the following research, each for different purposes.

Table 3 – Graphs Types according to Set Cardinality

Graph Type	Definition	Entities Modeled
Mixed	$G = (V, A, E)$	Non-symmetric road networks (with one-ways)
Undirected	$G = (V, A = \emptyset, E)$	Symmetric road networks (without one-ways)
Directed	$G = (V, A, E = \emptyset)$	Street networks Isosequential random graphs Isosequential highly-clustered graphs

The above mentioned definitions set out the general formal frame for all graphs considered in this research. All the different graphs types subsumed under equation 1 are usually defined independently of one another, if at all together (to our knowledge, the only examples of encompassing definitions for mixed, undirected and directed graphs are found in Stiege (2006) and Stiege (2007)). Despite this situation and given that the present research makes extensive use of all three graph types, such a general and all-encompassing set-theoretic definition has been here deemed useful, even necessary. Although other, more sophisticated graph types will be defined in the following sections, each of them will be defined as restrictions of geometric graphs as defined in equation 1; in other words, such graphs will be members of subsets of $\mathbb{G}$, as further membership conditions will be added to their intensional definition.

2.3.2 ARCS AND EDGES

Following the definition of graph superset $\mathbb{G}$, additional membership specifications are needed for arc and edge sets. For any graph G included in $\mathbb{G}$ with vertex set $V(G)$, the arc and edge sets $A(G)$ and $E(G)$ can be respectively defined as subsets of $\mathbb{A}(\mathbb{G})$ and $\mathbb{E}(\mathbb{G})$, whose intensions are defined in equations 2 and 3 [1].

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1. Given the inclusion of $A(G)$ and $E(G)$ in $\mathbb{A}(G)$ and $\mathbb{E}(G)$ respectively, $A(G) = \mathbb{A}(\mathbb{G})$ and $E(G) = \mathbb{E}(\mathbb{G})$ if and only if G is a complete graph

$$(2)\quad A(G) \subseteq \mathbb{A}(G) = \left\{ \langle v_1, v_2 \rangle \,\middle|\, \begin{array}{l} v_1, v_2 \in V(G) \\ v_1 \neq v_2 \\ \{v_1, v_2\} \notin E(G) \\ E(G) \neq \emptyset \rightarrow \neg(\langle v_1, v_2 \rangle \in A(G) \wedge \langle v_2, v_1 \rangle \in A(G)) \end{array} \right\}$$

$$(3)\qquad E(G) \subseteq \mathbb{E}(G) = \left\{ \{v_1, v_2\} \,\middle|\, \begin{array}{l} v_1, v_2 \in V(G) \\ v_1 \neq v_2 \\ \langle v_1, v_2 \rangle \notin A(G) \wedge \langle v_2, v_1 \rangle \notin A(G) \end{array} \right\}$$

Equations 2 and 3 specify the membership criteria for arcs and edges respectively. In both cases, the first criterium, $v_1, v_2 \in V(G)$, states that members must be pairs of vertices of G, $v \in V(G)$. The second criterium of both equations, $v_1 \neq v_2$, states that arcs and edges must be pairs of distinct vertices, thus excluding self-loops, that is, arcs or edges that link vertices to themselves. In the same vein, given that both $\mathbb{A}(G)$ and $\mathbb{E}(G)$ are not multisets, that is, sets in which members can appear more than once, equations 2 and 3 also rule out the possibility of parallel arcs and parallel edges, that is, of identical ordered and unordered pairs of vertices in $A(G)$ and $E(G)$ respectively. Given the exclusion of self-loops and parallel arcs or edges, all graphs considered in this research will be simple graphs.

The remaining criteria for arc and edge sets only apply to mixed graphs. The third membership criteria of equations 2 and 3 underline the incompatibility relationship characterizing arcs and edges over the same pair of vertices. As to the fourth and last condition for arc set membership, it also adds a further constraint for all mixed graphs used in this research, by eliminating the possibility of antiparallel arcs: if the edge set is non-empty and $\langle v_1, v_2 \rangle$ is an arc, then $\langle v_2, v_1 \rangle$ is not an arc. Obviously, this condition does not apply to directed graphs, as their edge set is empty. Given these three membership criteria for mixed graphs, any two vertices of mixed graphs can only be joined once. This type of configuration is essential to the adequate modeling of road networks, as two intersections can only be linked by one road, either a two-way or a one-way.

Given the above definitions of graph, vertex, arc and edge sets, the following terminology for vertices, arcs, and edges applies. An ordered pair of vertices $\langle v_1, v_2 \rangle$ constitutes an arc of graph G if and only if $v_1, v_2 \in V(G)$ and $\langle v_1, v_2 \rangle \in A(G)$. Likewise, an unordered pair of vertices $\{v_1, v_2\}$ is an edge of G if and only if $v_1, v_2 \in V(G)$ and $\{v_1, v_2\} \in E(G)$. Also, all arcs in $A(G)$ will by definition be ordered sets of vertices, such that $\langle v_1, v_2 \rangle \neq \langle v_2, v_1 \rangle$, while all edges $e \in E(G)$ will be necessarily unordered sets of vertices, $\{v_1, v_2\} = \{v_2, v_1\}$. The cardinality of $V(G)$ (i.e., the number of vertices) is referred to as its order and noted $|V(G)|$. In order to simplify the presentation of the algorithms described in the following sections, the set of links of G, $L(G) = \{l_1, ..., l_n\}$, referring to members of $A(G)$ or E(G) as defined by equation 4, will also be extensively used in this research:

(4) $$L(G) = (A(G) \cup E(G)).$$

Finally, two vertices $v_1, v_2 \in V(G)$ are said to be adjacent if they are both members of the same edge e or arc a, that is if $\{v_1, v_2\} = e \in E(G)$ or if $\langle v_1, v_2 \rangle = a \in A(G) \vee \langle v_2, v_1 \rangle = a \in A(G)$. In both cases, v_1 and v_2 are said to be incident to a and e respectively. Both members of a given edge e are called its ends, $end(e)$, while the first coordinate v_1 of an arc $a = \langle v_1, v_2 \rangle \in A(G)$ is called its tail, $tail(a)$, and the second its head, $head(a)$. Thus, an arc is said to be directed from its tail out and directed to its head.

To summarize what has been said thus far concerning vertices, arcs and edges, given any pair of vertices (v_1, v_2), five different linking scenarios are allowed by the above defined concepts and constraints. While the first three concern all graph types and imply at most one link, the fourth one applies to all graphs that are not directed (symmetric road networks) and implies one link, and the fifth one is only possible in directed graphs (street networks, isosequential random graphs and isosequential highly-clustered graphs) and implies two different links.

1. No arc or edge joins v_1 and v_2;
2. One arc directed from v_1 to v_2, $\langle v_1, v_2 \rangle \in A(G)$;
3. One arc directed from v_2 to v_1, $\langle v_2, v_1 \rangle \in A(G)$;
4. One edge joins v_1 and v_2, $\{v_1, v_2\} \in E(G)$;
5. Two antiparallel arcs joining v_1 and v_2, $\langle v_1, v_2 \rangle, \langle v_2, v_1 \rangle \in A(G)$.

This concludes the presentation of the fundamental graph-theoretic concepts used in the graph generation algorithms designed for the present research. However, additional formal definitions will be presented in the following chapters, in order to disambiguate mathematical concepts and relations pertaining to specific phases of the general methodological framework.

2.4 GEOINFORMATICAL PRELIMINARIES

As all road networks are embedded and develop in the "supra-haptic" geographic space mentioned in the introduction, their formal treatment falls within the field of geoinformatics, which "encompasses the acquisition and storing of geospatial data, the modeling and presentation of spatial information, geoscientific analyses and spatial planning, and the development of algorithms and geospatial database systems" (Ehlers, 2008, p.17). In particular, considerations relative to geographic data modeling, which "deals with the question of how the infinite complexity of the geographical world can be represented within a discrete, finite machine" (Goodchild, 1991, 0.195), are of crucial importance for the present purposes.

Geographic data models are mathematical constructs for representing real-world geographic objects or surfaces, for example buildings, roads, rivers, rainfall, soil types, and so on. Such models are used to perform spatial queries, analyses and produce cartographic maps (Hoel, 2008). As of today, geographic data models are based on two different conceptualizations of space: "as a set of continuous fields, and as a collection of discrete objects occupying an otherwise empty space" (Goodchild, 2008, p.202).

> Discrete phenomena are usually objects that can be directly recognized due to the existence of their geometrical boundaries with other objects, such as a road, a lake, and a building. Continuous phenomena usually do not have observable boundaries are vary continuously over space, such as temperature, air quality, and reflected solar radiation from land surfaces (Yang and Di, 2008, p.356).

These two different conceptualizations of space, spatial entities and phenomena translate into two different geographical data modeling, display and storage models: raster maps and vector maps.

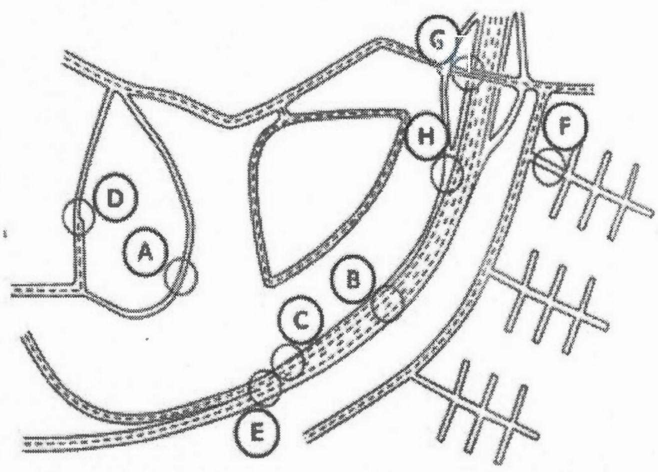
Raster models store, process and display geographic data and features in matrix form, that is, into a regular grid of cells. By defining space as a giant table composed of (rectangular or square) equally sized cells arranged in rows and columns, the spatial location of each cell as well as its contents is implicitly contained within matrix ordering (Goodchild, 2008; Lim, 2008; Yang and Di, 2008). Given this, a raster map can be build, by assigning a specific color to each grid cell. The resolution of such map can be coarse- or fine-grained: depending on matrix size, each cell can represent a spatial area ranging from centimeters to kilometers. Additionally, rastering can also allow for data storage, as each cell can be assigned a series of attribute values averaged over the spatial area it represents.

On the other hand, vector data models involve the representation and storing of data as discrete geometric primitives such as points, lines or polygons. Embedded in two-dimensional space through coordinate pairs, point features can stand for specific geographical entities (for example, cities or landmarks) or constitute boundaries (limits) of linear features. Including both line and polyline (a series of adjacent lines considered as a single geographical feature) features, such linear features can also stand for distinct geographical entities (like rivers or roads) or constitute boundaries of polygon features, whose form can stand for geographical entities such as buildings or countries. Attributes are associated with each feature, each attribute having the same value within the feature geometric boundaries, given it is considered a single discrete object. Moreover, different vector models can be built, depending on the level of topological coercivity, that is, on the quantity of rules determining how the coordinates of coincident points, lines and polygons are stored. Spaghetti models are the most simple ones, the topological relationships among geometrical features of different types, for example their adjacency, boundary sharing, intersection or overlapping, is not made explicit at all, hence their denomination. Network models, which consist of one-dimensional collections of topologically interconnected point and line features, are for their part mainly used to represent reticular structures inherent in different geographical systems, for example transportation infrastructures and drainage systems. Topologically speaking, these models are more constraining than spaghetti models, as lines and polylines are necessarily delimited by and intersecting at point features. Finally, topological models constitute the most elaborate and constraining vector models. In addition to representing given geographical entities, point and line features also serve in these models to define and delimitate faces and polygon features, which can stand for geographical entities such as buildings, land areas and countries. Topological models are also used to define and enforce data integrity rules. These rules, including the non-overlapping of polygon features and the boundary-sharing of adjacent geometric features (polylines connecting at endpoints, adjacent polygons sharing a common line), ensure the topological consistency of the database and allow for a more realistic representation of geographic features (Hoel, 2008; Gandhi, 2008; Yang and Di, 2008).

Vector models, by allowing effective geometric representation of geographical features at different scales, prove ideal for cartographical purposes and applications. But also, given their geometrically discrete character, which structurally distinguishes them from raster models, vector models are often used for specific types of geographical entities, phenomena and analysis. Indeed, while raster data is generally more suited for environmental modeling and research, vector data is more appropriate for the representation and analysis of human-related activities, phenomena and geographical objects. As human-made transportation infrastructures, road networks are no exception to that rule; even though raster-based modeling of road networks is possible, their cartographic representation and spatial analysis constitute exemplary cases of network-style, vector-based geographic modeling.

A road network data model enables the modeling of pertinent aspects of a road network infrastructure. Figure 3 offer a fairly representative sample of some of the possible road types and configurations that may be part of a ground transportation infrastructure: single, one-way roads (A), bi-directional roads with multiple lanes in both directions (B), beginning of new road lanes (C), bi-directional roads that abruptly change into one-way roads (D), road splittings (E), restriction-free roads in residential areas (F), restricted U-turns as well as restricted line changes (H).

Figure 3 – Example of different road configurations (Speicys and Jensen, 2008, p.973)



Despite the diversity of such configurations, their geometrical representation in vector data models is rather straightforward. Coordinate points are used in order to represent the centerline part of each lane or road; distance between points is kept proportional to the distance between the corresponding real-world locations. Pairs of connected points define line segments that model sections of lanes or roads. Road segments are thus modeled using either a single line connected by two points, if the corresponding road section is rectilinear, or a polyline made of a sequence of alternating points and straight lines, in order to adequately represent curvilinear road features (Speicys and Jensen, 2008). Given that the configuration of the point features in the vector map is isomorphic to the relational structure of the real-world location to which they correspond, the length and configuration of linear features delimited by these coordinate points is also isomorphic to the length and configuration of the corresponding road sections (George and Shekar, 2008).

CHAPTER III

FROM VECTOR MAPS TO SYMMETRIC AND NON-SYMMETRIC ROAD AND STREET NETWORKS: METHODOLOGICAL AND ALGORITHMIC CONSIDERATIONS

The main objective of the present research is to assess the impact of road direction on the street-based modeling of road networks through calculation, analysis and comparison of the ω scores of symmetric and non-symmetric street networks. In achieving this aim, different implementation phases need to be distinguished and executed:

1. Vector Map Collection and Preprocessing
2. Conversion of Vector Maps into Planar Straight Line Graphs (Road Networks)
3. Construction of Path Partitions of Planar Straight Line Graphs
4. Generation of Intersection Graphs of Path Partitions of Planar Straight Line Graphs (Street Networks); calculation of Average Clustering Coefficient and Characteristic Path Length of Intersection Graphs
5. Generation of Isosequential Random Graphs; calculation of Characteristic Path Length of Isosequential Random Graphs
6. Generation of Isosequential Highly-Clustered Graphs; calculation of Characteristic Path Length of Isosequential Highly-Clustered Graphs
7. Calculation of ω scores for Street Networks.

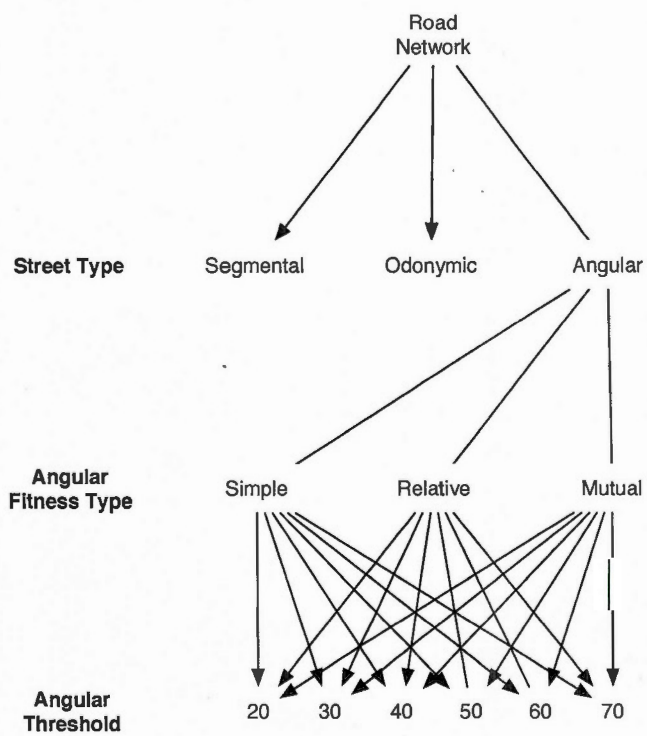
While the first three phases are treated in distinct sections, all the other stages are dealt with in the last section of the chapter, as they all imply operations on directed graphs. Following the first data collection and preprocessing phase, vector maps are converted into road networks, which are planar straight line graphs in which angles and distances are respectively kept identical and proportional to those of the corresponding vector maps. Two types of road networks are generated, depending on road direction: if the latter is taken into account, non-symmetric road networks are created through generation of mixed planar straight line graphs; otherwise, symmetric road networks are build using undirected planar straight line graphs. This procedure results in the generation of one symmetric road network and one non-symmetric road network for each urban area modeled. In the present research, the London neighbourhoods of Barnsbury, Clerkenwell, and Kensington are considered, which implies the generation of six different road networks, as shown in table 4.

Table 4 – Road Networks generated in this research

neighbourhood	Direction type
Barnsbury	Symmetric Non-Symmetric
Clerkenwell	Symmetric Non-Symmetric
Kensington	Symmetric Non-Symmetric

Thirdly, in order to recover or build the different streets contained in the generated symmetric and non-symmetric road networks, path partitioning of the latter are carried out on the basis of various parameterization settings. The first partition parameter is street type: in the present research, three different street types are proposed, namely segmental streets, odonymic streets and angular streets. Segmental street paths are formed by including in the same street adjacent road sections comprised between two intersections. As for odonymic street paths, they are built by joining adjacent road sections which have the same odonym (street name). Finally, angular street paths are formed through the merging of adjacent road sections whose deflection angle satisfy specific angularity requirements. The conditions pertaining to deflection angles constitute additional parameters, specific to angular path partitions. The first angular parameter, angular fitness type, refers to different requirements adjacent road sections must satisfy in order to be added to a given street path. Three such fitness types are distinguished: simple fitness, relative fitness and mutual fitness; each fitness type will be defined in section 3.3.4.3. Finally, angularity threshold constitutes the upper bound for angular street path formation: two adjacent road sections forming a deflection angle whose value is greater than the angular threshold given as input cannot be part of the same angular street path. Six different threshold values have been selected in this study: $20^o, 30^o, 40^o, 50^o, 60^o, 70^o$. In light of this, 20 different parameter configurations are possible, as shown in Figure 4; since one path partition is carried out for each distinct parametrization setting of each different road network, a total of 120 different path partitions are thus generated.

Figure 4 – Path Partition Parametrization Settings



Finally, in order to evaluate the small-worldness of symmetric and non-symmetric street networks, three different types of directed graphs are generated. First of all, symmetric and non-symmetric street networks are obtained by generating intersection graphs for all path partitions of symmetric and non-symmetric road networks; their respective average clustering coefficient and characteristic path length are recorded for further ω score analysis. Following this, 10 isosequential random graphs and 10 isosequential highly-clustered graphs (with identical in-degree and out-degree sequences) are generated on the basis of the in-degree and out-degree sequences of each street network; then, the mean characteristic path length and mean average clustering coefficient of those isosequential graphs are respectively recorded. Following generation of all these directed graphs, ω score are calculated for each street network; results, analysis and comparison of these scores is given in chapter IV.

In order to facilitate comprehension of the methodological phase of this research, algorithms written in set-theoretic pseudocode and corresponding to the different vector- and graph-processing algorithms designed by the author are presented and described in the following sections.

3.1 DATA COLLECTION AND PREPROCESSING

For this research, the road networks of the London neighbourhoods of Barnsbury, Clerkenwell and Kensington have been considered. This decision is consistent with much of the research done in the Space Syntax literature, in which these neighbourhoods were often investigated (Hillier et al., 1993; Jiang, 2009a; Jiang and Jia, 2011). As in previous studies, all three neighbourhoods slightly overlap on each other, in order to alleviate edge effects [1]. GIS data for these three neighbourhoods is of

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1. Edge effects refer to structural changes that occur near imposed, artificial boundaries of otherwise continuous or unbounded spatial objects (Fotheringham and Rogerson, 1993), changes which result from the ignorance of all cross-boundary relations and objects (Griffith, 1980, 1983; Griffith and Amrhein, 1983; Griffith, 1985). One of the fastest and most efficient ways to counter such effects for a given spatial region is to work with expanded coverage areas. For example, it has been suggested that measures of urban sprawl should also take into account interdependencies and interactions with neighbouring rural

OpenStreetMap (OSM) format (.osm), a Volunteered Geographical Information (VGI) project aiming at generating an open-source, user-generated, and vector-based representation of the geographic world.

OpenStreetMap is probably the most extensive and effective "crowdsourced" or Volunteered Geographical Information (VGI) project currently under development, with 1,654,095 users registered as of June 4th, 2014 (OpenStreetMap, 2013). Born at University College London in July 2004, "OSM follows the peer production model that created Wikipedia; its aim is to create a set of map data that's free to use, editable, and licensed under new copyright schemes" (Haklay and Weber, 2008, p. 13):

> The database is built by contributors, usually called mappers with OpenStreetMap, who gather information by driving, cycling, or walking along streets and paths, and around areas recording their every move using Global Positioning System (GPS) receivers. This information is then used to create a set of points and lines that can be turned into maps or used for navigation. (...) Other data is gathered from out-of-copyright maps, public domain databases (ones with no copyright protection), or in some cases donations of proprietary databases by the companies that own them. (...) The database uses a wiki-like system where any mapper can add or edit any feature of the area, and a full editing history is kept for every object. This means any mistakes or deliberate vandalism can be rolled back, keeping the data accurate. OpenStreetMap doesn't use an existing geographic information system (GIS) to store its data, but instead uses its own software and data model to make the crowdsourcing process as easy as possible, and to allow the maximum level of flexibility in what gets mapped and how (Bennett, 2010).

As GPS measurements are all based on the 1984 edition of the World Geodetic System (WGS-84), the OpenStreetMap database is build on this same geographic coordinate system. Concerning OSM data per se, many studies have already shown that its pre-

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areas (Theobald, 2001). Now, in studies focusing on multiple adjacent areas, which is the case here, coverage area expansion results in mutual overlapping of the areas under study.

cision, completeness and overall quality is surprisingly good (Girres and Touya, 2010; Kounadi, 2009; Mooney and Corcoran, 2012; Neis et al., 2011; Ueberschlag, 2010; Zielstra and Zipf, 2010). Regarding the city of London in particular, Ather (Ather, 2009) and Hakley (Haklay, 2010) have compared OSM data with the Meridian 2 dataset from the Ordinance Survey (OS), Great Britain's national mapping agency. Their research has shown that, for Central London as well as for the centers of England's biggest cities, OSM data is of very good positional accuracy and mapping completeness: on average within 6 m of the position recorded by the OS, making it a comparable and viable alternative to professional surveys. As for street names, it has been shown in (Ather, 2009) that, through analysis of 98.53 km of road in the Central London area, less than 5 km (5% of total length) were unlabeled, while naming accuracy has been shown to be above 90%, with most errors relating to minor issues such as punctuation, road type or spelling (Pitsis and Haklay, 2010). Hence, the odonym completeness and accuracy degree are both very high.

In these cases, datasets used for quality analysis are at least 4 years old. Hence, given that the updating rate of OSM data is very high and that London "has received more attention from OSM participants than any other region of the world" (Haklay, 2010, p. 684), we can easily suppose that the accuracy, completeness and overall quality of OSM geographical and attribute data for this city is even better today than it was at the time of these studies.

The OSM dataset used for this research (London.osm.shp.zip) has been downloaded on March 30th, 2013, and is based on road information available on the `BBBike.org` website (`http://goo.gl/LHcBlj`). The dataset includes a set of thematic layers (buildings, points, railways, roads, and waterways), classified according to OSM tags. For the present research, however, only the "road" layer was used: the advantage of this specific dataset layer over the several other OSM road vector maps available lied on the "one-way" tag it contained, which conveyed all the road direction information necessary for the identification of one-ways and the realization of this research. As was said earlier of vector data models in general, each attribute has the same value for the whole feature, which means that different attribute values necessarily imply different vector features. This of course is also true of the "one-way" attribute: two adjacent roads sections that have different one-way attribute values are necessarily represented by distinct vector features. This attributive homogeneity of vectors has important implications for the present study, especially for street recovery procedures. More will be said on that matter in section 3.3.3.

In order to better meet the needs of the present research, the collected dataset however required some preliminary manipulation. First, OSM files constitute complete topological vector data models, including vector layers composed of point features representing different landmarks such as bus and tram stops as well as polygon features corresponding to geographic entities such as buildings or underground stations. Given the present research objectives, layers relating to point polygon features have been deemed superfluous and thus excluded from the corresponding vector maps. Moreover, since only road-related polyline features are considered in this research, geographic and polyline-based entities such as underground tunnels, electric lines and waterways were also excluded from the vector map. These different filtering operations resulted in the conversion of the different OSM files from topological data models to strict road network data models.

Filtering of road-related features has also been necessary, as roads rarely used or unused by motorized vehicles had to be excluded. Vector features pertaining to the OSM highway types specified in table 5 were thus excluded:

In addition to this, geographical entities with unorthodox OSM highway tags ('proposed', 'no', 'depot', 'crossing', 'conveyor', 'elevator') were considered as anomalies and excluded from the vector map, which resulted in the deletion of a little more than a dozen vectors. In the course of geometrical and topological graph creation, proof-checking and editing of road direction has also been done. Finally, due to the asymmetric nature of certain roads and in order to allow for free-flowing human movement through the road networks, neighbourhood boundaries as fixed in Jiang and Liu (2009) have been slightly modified in order to allow for the generation of strongly connected directed and mixed graphs, in which there is a path from each node to each other node

Table 5 – List of excluded vector features

Highway Type	Number of features	Description
Pedestrian	1824	For roads used mainly/exclusively for pedestrians in shopping and some residential areas which may allow access by motorized vehicles only for very limited periods of the day
Track	78	Roads for agricultural or forestry uses etc. Often rough with unpaved/unsealed surfaces
Service	5648	For access roads to, or within an industrial estate, camp site, business park, car park, etc.
Path	380	A non-specific or shared-use path
Footway	12262	For designated footpaths; i.e., mainly/exclusively for pedestrians
Cycleway	615	For designated cycleways; i.e., mainly/exclusively for bicycles
Bridleway	63	For horses (in the UK, these are rights of way for pedestrians, horse-riders and cyclists)
Steps	936	For flights of steps (stairs) on footways
Proposed	2	For planned roads
Construction	28	For roads under construction

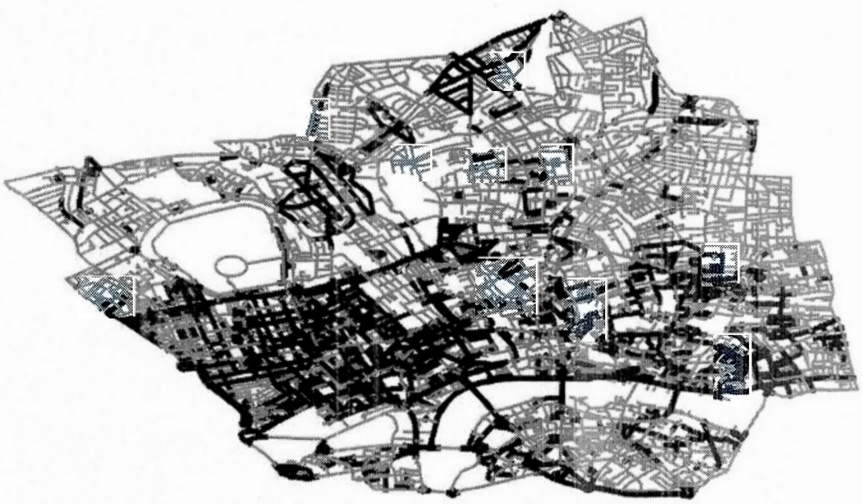
(for more information, see definitions of connectedness and strong connectedness in section 3.3.1).

These different editing operations resulted in the maps shown in Figures 5, 6 and 7, which distinguish one-way road sections (black) from two-way ones (grey).

Finally, the different vector maps used in this study have been planarized by cutting all polylines at intersections. This kind of operation is usually not recommended, as planarity can be violated due to bridges that cross above perpendicular features. However and as in (Lämmer et al., 2006), given the relatively low number of bridges in the three neighbourhoods under study, the loss of topological information related to pla-

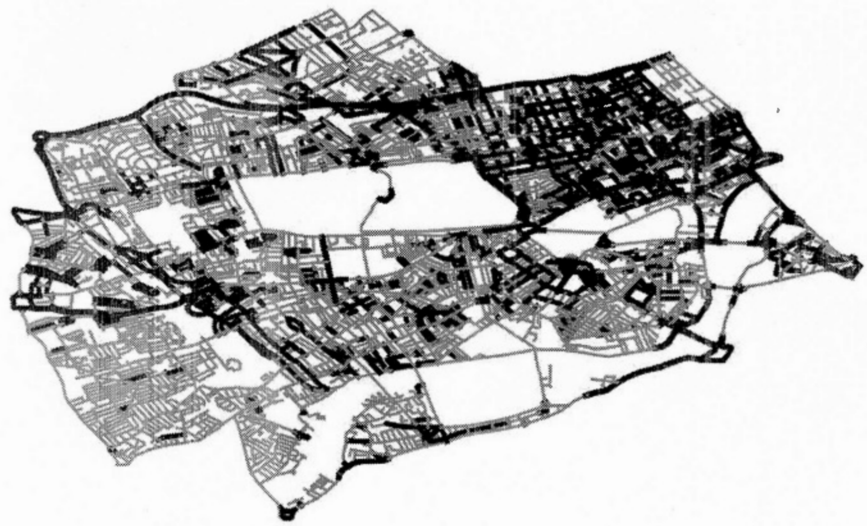
Figure 5 – Map of Barnsbury

Figure 6 – Map of Clerkenwell



narization can be considered minor. Moreover, in the case of OSM maps, the length and configuration patterns of vector features is rather conjectural, as it often reflects the particular needs, availability or itinerary of the contributors; given that the aim of

Figure 7 – Map of Kensington



the present research consists precisely in determining the number and configuration of the different streets that can be built out of the road networks under investigation, planarization of all road features thus constitutes an essential preprocessing operation.

3.2 PLANAR STRAIGHT LINE GRAPHS

In order to carry out the graph-theoretical operations and analyses needed for the present research, road vector maps must be converted at the outset into structurally equivalent graph-theoretic structures, here called "road networks". This metrically-faithful conversion of vector maps into road networks can be made possible with the use of planar straight line graphs.

Due to their planar nature, planar straight line graphs have the property of being embeddable in the plane in such a way that no two arcs or edges intersect except at a common endpoint. Using Kuratowski's theorem (Kuratowski, 1930), planar graphs can be defined by means of a forbidden graph characterization, that is, by specifying which type of graph is excluded from the defined set. According to this theorem, a graph is said planar if it doesn't contain subgraphs homeomorphic to the complete 5-graph or the complete bipartite 3-graph.

In order to fully understand Kuratowski's theorem, clarifications regarding subgraphs, complete graphs and homeomorphisms seem here necessary. A graph $G' = \{V', A', E'\}$ is a subgraph of a graph $G = \{V, A, E\}$ and G a supergraph of G' if all vertices, edges, and arcs of G' are contained in G. Also, a simple undirected graph is said complete and denoted K_n if all n vertices of the graph are pairwise adjacent, that is, if every pair of vertices in the graph are joined by an edge, which results in a total of $n(n-1)/2$ edges. Thus, the complete 5-graph, also referred to as K_5, contains 5 vertices and 10 edges connecting every vertex to all the other vertices.

On another front, a graph G is said bipartite if it contains a partition of vertices in two disjoint subsets p_1 p_2, such that each vertex of p_1 is connected only to vertices of p_2 and reciprocally. In other words, each edge of a bipartite graph has ends in both p_1 and p_2, and no edge join vertices from the same subset. A complete bipartite graph, denoted $K_{m,n}$, represents a special kind of bipartite graph in which all m vertices of subset p_1 are connected to all n vertices in p_2 and only those vertices. Given this definition, the complete bipartite 3-graph or $K_{3,3}$ is a bipartition of vertices in 2 disjoint subsets or parts, in which edges connect each one of the three vertices in p_1 to all three vertices in p_2.

As regards to homeomorphism, two graphs G and H are homeomorphic and noted $G \sim H$ if one can be obtained from the other by subdividing at least one edge e into multiple adjacent edges through the addition of vertices between the endpoints of e and incident to it [2].

To resume, planar graphs can thus be defined as graphs that do not contain subgraphs

2. Edge subdivision can also be defined as to the replacement of an edge $\{v_1, v_2\}$ by a v_1, v_2-path of any given length.

homeomorphic to K_5 or $K_{3,3}$. Stated as such, both Kuratowski's theorem and the definition of planar graphs can't be applied to graphs whose arc set is nonempty. However, this situation may be remedied by appealing to the concept of underlying graph. The underlying graph of a directed or mixed graph G, U(G), is the undirected graph obtained by replacing each arc or pair of antiparallel arcs between two vertices $v_1, v_2 \in V(G)$ by an edge $\{v_1, v_2\} \in E(G)$. Given this definition, underlying graphs have the same incidence structure as their mixed or directed correlates; in fact any orientation or partial orientation of undirected graphs, that is, any integral or partial replacement of edges by arcs, preserves the original incidence structure of the undirected graph. It therefore follows that directed or mixed graphs are planar if their underlying graph does not contain a subgraph homeomorphic to K_5 or $K_{3,3}$.

(1) $$PG = \{x \mid x \in \mathbb{G} \wedge \forall H((H \sim K_5 \vee H \sim K_{3,3}) \rightarrow (U(x) \not\supseteq H))\}$$

Given the above mentioned considerations, a planar straight line graph is a plane graph with straight edges or, in other words, a graph in which curvilinear edges are subdivided in order to form a sequence of straight adjacent lines. A theorem by Istvan Fàry (Fary, 1948) states that every planar graph has a straight line drawing; in other words, the curved edges of a planar graph can be replaced by a sequence of adjacent straight lines without altering the graph's planarity. Also, following the above-mentioned definition of homeomorphism, a planar graph is homeomorphic to its planar straight line counterpart. Given all this, planar straight line graphs do allow for the graph conversion of geographic vector databases.

The vector maps of the three London neighbourhoods have been converted into graphs using NetLogo, an agent-based simulation platform developed by the Centre for Connected Learning at Northwestern University and endowed with very efficient GIS and network extensions of (Wilensky, 1999). One of the main reasons behind the use of NetLogo is the ease with which ESRI shapefiles can be imported into the platform and mapped into the NetLogo world. The NetLogo world is based on a cartesian coordinate system, endowed with X and Y axis of adjustable length; for the present research, a NetLogo space with grid size 66×44 has been chosen. Conversion of OpenStreetMap WGS-84 coordinates to NetLogo coordinate space can be done by downscaling the size of the vector map minimum bounding rectangle (MBR), i.e. the smallest rectangle that can contain all features of the road network, in order to fit NetLogo spatial requirements.

Algorithm 1 describes the procedures designed for the conversion of vector-based road networks to Planar Straight Line Graphs (PSLG). In order to convert the vector maps into distance-proportional and angular-identical geometrical graphs, points are converted into graph nodes, and for every line delimited by 2 points, a link is created between the corresponding nodes. Two different geometry types are used, undirected and mixed, depending whether or not road asymmetry is taken into account. Mixed geometrical graphs thus contain two types of connections, undirected and directed links, standing respectively for two-way and one-way road sections.

The data structure of vector models corresponds to a series of nested lists: the global list contains all the polyline features of the road network; each polyline feature is itself a list of coordinate pairs, which refer to the location of the different point features connecting the lines composing the polyline feature and defining its geometric trajectory. In set-theoretic terms, a vector map is thus an ordered set of ordered sets (polyline features) of ordered pairs (point features). In this sense, the extraction of all the information necessary for the generation of an isomorphic PSLG constitutes a rather standard list-processing task.

Thus, for each point feature (line 7) of each polyline feature in the vector map (line 4), if the coordinates of that point feature do not already correspond to a vertex in $V(PSLG)$ (line 11), a new vertex is added to the vertex set of *PSLG*, having for coordinates the same coordinates of that point feature (line 12). Moreover, if the point feature in

Input: shapefile vector map (*.shp*)
Output: *PSLG*

```
begin
    i = 0;
    j = 0;
    while i < length(LinearFeatures) do
        thisLinearFeature = LinearFeatures(i);
        thisName = name(thisLinearFeature);
        while j < length(thisLinearFeature) do
            thisCoordinatePair = thisLinearFeature(j);
            x1 = thisCoordinatePair(0);
            y1 = thisCoordinatePair(1);
            if ⟨x1,y1⟩ ∉ V(PSLG) then
                V(PSLG) = {V(PSLG) ∪ ⟨x1,y1⟩};
            if j ≥ 1 then
                PreviousCoordinatePair = thisLinearFeature(j − 1);
                x2 = PreviousCoordinatePair(0);
                y2 = PreviousCoordinatePair(1);
                if GraphType=Mixed and OneWay(thisLinearFeature)="True" then
                    A(PSLG) = {A(PSLG) ∪ ⟨⟨x2,y2⟩, ⟨x1,y1⟩⟩};
                    label ⟨⟨x2,y2⟩, ⟨x1,y1⟩⟩ = thisName;
                else
                    E(PSLG) = {E(PSLG) ∪ {⟨x2,y2⟩, ⟨x1,y1⟩}};
                    label{x2,y2,x1,y1} = thisName;
            j = j + 1;
        i = i + 1;
```

Algorithm 1: Planar Straight Line Graph Generation Algorithm question isn't the first point feature of the polyline it is included in (line 13)[3], a link from it to the previous point feature of the polyline is added to $PSLG$. Such linking is of course direction-sensitive: in non-symmetric context, the polyline feature in question is a one-way, then an ordered pair having respectively the previous and current point features as first and second coordinates is added to the arc set $A(PSLG)$ (line 17); on the other hand, in symmetric context or if the polyline feature is not a one-way, the graph to be generated is undirected, an unordered pair having the previous and current point features as ends is added to the edge set $E(PSLG)$ (line 21).

Following the creation of a link, each arc or pair thus created is given a label corresponding to its odonym or street name (lines 19,22). Usually, an edge-labeling of a graph G corresponds to an assignment of integers to the arcs or edges of G, given certain conditions (Gallian, 2013). The NetLogo platform however allows strings of characters to be implemented as arc and edge labels, thus allowing for the assignation, to all arc and edges, of their respective odonym. Still, an integer-based labeling would also be possible, by creating a list of all odonyms for each street network and assigning to each arc or edge the index number of the odonym of the road section it represents.

Following execution of this algorithm, a planar straight line graph is generated, geometrically isomorphic to the vector map given as input. This structural isomorphism rests on the following mapping: every point feature corresponds to an embedded graph node, every line between two points corresponds to an arc or an edge joining two nodes. The only problem is that such mapping does not take polylines into account: every graph arc or edge corresponds to a line of the vector map, but nothing in planar straight line graphs can be formally equated with series of adjacent line features forming autonomous polyline features. In the perspective of the present research, this means that planar straight line graphs cannot by themselves allow for the formal representation of streets, considered as ordered sets of arcs or edges. To address this shortcoming, path partitions on planar straight line graphs are to be generated. This procedure, which will

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3. In set theory, such condition is unnecessary, given that the extensionality axiom precludes multiple membership for sets. However, in NetLogo, multiple entities can have the same coordinates, hence the necessity to add only vertices with unprecedented coordinates

be described and discussed in the following chapter, will allow for the construction of undirected and directed, arc-or-edge-disjoint, paths on PSLGs. As it will be shown, being ordered sets of arcs or edges, such paths are not only formally similar to polylines but can also, taken altogether and given specific path construction instructions, allow for the "recovery" of the street structure of any given road network.

3.3 STREET PATHS AND PATH PARTITIONS

Paths are one of the most fundamental concepts of graph theory, being rigorously defined and extensively discussed by any introductory manual in the field. Derived from the concepts of walk and trail, their importance for the present research is fundamental, as the process of recovering a street from a series of adjacent road sections is here translated into a path construction problem, while the recovery of a whole street structure from a given underlying road network formally amounts to the construction of a specific path partition of the corresponding planar straight line graph. Following the definition of paths and path partitions on graphs, the present section then presents and describes the different algorithmic procedures used for the generation the 120 different path partitions of the 6 undirected and mixed planar straight line graphs (that is, the 6 symmetric and non-symmetric road networks) considered for the present research.

3.3.1 WALKS, TRAILS AND PATHS

Given a Graph G, a mixed walk is a finite incidence sequence of alternating vertices and links[4], beginning with initial vertex v_1 and ending with terminal vertex v_n, such that $W_G(v_1,v_j) = v_1, l_1, v_2, \ldots, v_{n-1}, l_{n-1}, v_n$. All vertices included in the walk sequence that do not correspond to the initial or terminal vertices are called inner vertices. The length $||W_G(v_1,v_n)|| = (n-1)$ of a walk corresponds to the number of links in that walk sequence.

4. Links were defined in section 2.3.2 as referring either to arcs or edges.

Since by definition, links in mixed walks can be either arcs or edges, different link crossing scenarios and walk types are possible (Stiege, 2007). As regards to link crossing, while edges can be traversed in both directions due to their symmetric nature, three different arc crossing scenarios are possible:

1. forward crossing: arcs can only be traversed from tail to head
2. backward crossing: arc can only be traversed from head to tail
3. any crossing: arc can be traversed in both directions

Given the intimate link between one-way and two-ways roads, on the one hand, and motorized vehicle movement and overall traffic flow on the other, only forward arc crossing is allowed in this research. The reason for such restriction is pretty straightforward : by definition, one-ways allow vehicular movement in only one direction, direction which always correspond to tail-to-head arc traversals in planar straight line graphs. Apparently obvious, this tail-to-head arc traversal restriction however has huge structural implications on the resulting path partitions and street networks; in fact, the impact of road asymmetry on street structure emerges from this very restriction.

Also, due to the mixed nature of some planar straight line graphs, two additional walk types need to be defined, namely directed and undirected walks, which respectively and exclusively contain arcs and edges. Thus, given a mixed or directed graph G, a directed walk is a finite incidence sequence of alternating vertices and forward-crossed arcs, beginning with initial vertex v_1 and ending with terminal vertex v_n, such that $W_G(v_1, v_n) = v_1, a_1 = \langle v_1, v_2 \rangle, v_2, ..., v_{n-1}, a_{n-1} = \langle v_{n-1}, v_n \rangle, v_n$. Likewise, given a mixed or undirected graph G, an undirected walk is a finite incidence sequence of alternating vertices and edges, such that $W_G(v_1, v_n) = v_1, e_1 = \{v_1, v_2\}, v_2, ..., v_{n-1}, e_{n-1} = \{v_{n-1}, v_n\}, v_n,$. Given that vertices included in walks are already members of the relevant links and that incidence is already implied in the walk sequence, each vertex included in an ordered or unordered pair being also present in the following or preceding link, inclusion of vertices in walks sequences is redundant. Directed and undirected walks can thus be defined simply as sequences of adjacent arcs and edges respectively; thus,

$W_{A(G)}(v_1,v_n) = \langle v_1,v_2\rangle,...,\langle v_{n-1},v_n\rangle$ and $W_{E(G)}(v_1,v_n) = \{v_1,v_2\},...,\{v_{n-1},v_n\}$.

Given two vertices $v_1, v_2 \in V(G)$, vertex v_2 is said to be reachable from vertex v_1 if G contains a walk from v_1 to v_2. An undirected graph is connected if every vertex is reachable from every other vertex. Likewise, mixed and directed graphs are strongly connected if every vertex is reachable from every other vertex; in other words, mixed and directed graphs are strongly connected if, for any pair of vertices $v_1, v_2 \in G$, G contains both $W_{A(G)}(v_1,v_2)$ and $W_{A(G)}(v_2,v_1)$. In the present research, given that road networks allow traffic from everywhere to everywhere else, undirected graph connectivity and mixed or directed graph strong connectivity are a inescapable outcome of any thorough modeling process. In fact, given that the main function of roads is to give access to other roads and places, a disconnected road serves no purpose at all.

Any mixed, undirected or directed walk with distinct links (arcs or edges) is called a trail. In the same vein and most importantly, a trail with distinct inner vertices is called a path. Paths can be closed or open, depending whether their initial and terminal vertices are identical or not; closed paths are also known and referred to in the literature as cycles. Since all inner vertices, arcs and edges included in paths are distinct, paths can thus be defined as ordered sets of unordered or ordered pairs, the starting and terminal vertices of a path corresponding respectively to the tail of the first element and the head of the last member of the path[5]. Thus, an undirected path on G, $P_{E(G)}\langle v_1 - v_n\rangle = \langle\{v_1,v_2\},...,\{v_{n-1},v_n\}\rangle$, is defined here as a sequence of edges such that every edge in the tuple has an end in common with both the preceding and next members of the tuple. Similarly, a directed path $P_{A(G)}\langle v_1 - v_n\rangle = \langle\langle v_1,v_2\rangle,...,\langle v_{n-1},v_n\rangle\rangle$ is a tuple of arcs such that the head of one arc in the tuple is the tail of the next member of the tuple. Given these definitions, two directed paths are arc-disjoint if they do not have any arcs in common, while two undirected paths are edge-disjoint if they do not have any edges in common. Arc-disjoint directed paths and edge-disjoint undirected path are of crucial importance for the definition and understanding of path partitions, to which we now

5. Properly speaking, walks are sequences but not sets, given that vertices, arcs and edges can appear multiple times in them. Walks can however be defined as multisets, which allow multiple membership of elements, but such definition has been deemed unnecessary for the present purposes

turn.

3.3.2 PATH PARTITIONS

In combinatorics and algorithmics, problems and research relating to path partitions [6] of graphs have a long history and are still today the object of scientific enquiry, having found various applications in fields as diverse as networking, block designs, bioinformatics and software testing (Arumugam et al., 2013; Ntafos and Hakimi, 1979).

Defined summarily, a path partition of a graph G is a partition in which every component (or part) is a path. As with partitions in general, a given set is amenable to an extremely high number of different path partitions, growing exponentially with set cardinality. In combinatorial mathematics, Bell numbers (B_n) are used to count the number of possible partitions of a set of n objects. Bell numbers can be defined as a sum of Stirling numbers of the second kind, written $\left\{ {n \atop k} \right\}$, which count the number of partitions of a n-element set into k nonempty subsets:

(2)
$$B_n = \sum_{k=0}^{n} \left\{ {n \atop k} \right\}.$$

According to this equation, the first 10 Bell numbers are as follows:

---

6. In mathematical literature, such path partitions have also been called path separations (Balogh et al., 2013; Falgas-Ravry et al., 2013), path decompositions (Alspach and Pullman, 1973; Arumugam et al., 2013; Heinrich, 1993) or path covers (Diestel, 2000; Franzblau and Raychaudhuri, 2002). In the present research, however, the expression "path partition" will be privileged over all others, given its obvious and direct set-theoretical rendering.

$$\langle 1,1,2,5,15,52,203,877,4140,21147,...\rangle.$$

However, given that paths are ordered sets, path partitioning represents a permutational operation that results in "ordered partitions" (Stanley, 1997, p.297), not unordered ones. Thus, the number of possible path partitions of a set correspond to the Ordered Bell number or Fubini number of this set, not its standard Bell Number. Written $a(x)$, Fubini numbers can also be formulated using Stirling numbers of the second kind:

(3) $$B_n = \sum_{k=0}^{n} k! \left\{ {n \atop k} \right\}.$$

And as the first 10 Fubini numbers show, the growth rate is steeper than for unordered partitions:

$$\langle 1,1,3,13,75,541,4683,47293,545835,7087261,...\rangle.$$

Ordered partition problems can thus quickly become complex or intractable. Imposition of certain path specifications or distinctions helps reduce the number of possible path partitions of a given set (Arumugam et al., 2013). Two such distinctions are particularly interesting for the present research. First, paths can either be directed or undirected; while this implies that any undirected, directed or mixed graph can be partitioned, studies involving partitions of both directed and undirected paths seem to be lacking. Another major partitioning distinction relates to vertex and link path membership: vertex-disjoint path partitions are such that no vertex appears in more than one path; similarly, in the case of link-disjoint path partitions, the arc or edge set of a graph is partitioned in such a way that every arc or edge is a member of only one path. Finding sets of vertex-disjoint or edge-disjoint paths in graphs "is one of the most ancestral and most studied themes of graph theory" (Naves and Sebõ, 2009, p. 261), with important applications in domains such as the Very-Large-Scale Integration (VLSI) of electronic chips (Korte et al., 1990).

Vertex-disjunction however represents a too strong constraint for the present research, as street paths may share vertices (corresponding to intersections in real-world road networks). On the other hand, arc- and edge-disjunctions are both necessary: while all road sections are part of a given street, no road section is part of two different street paths, and such condition also applies to real-world road networks. Issues regarding edge-disjoint path partitions of graphs count among the most active areas of investigation in graph theory. One the main focal points of investigation regarding such partitions relates to the problem of finding "the minimum number of edge-disjoint paths covering the edges of G" (Donald, 1980, p.189), also known as the path number problem. While it seems obvious that every non-trivial or non-empty graph allows for an edge-disjoint path partition (Arumugam et al., 2013), the problem of finding the least number of paths that can cover a graph has proven to be NP-complete for both undirected and directed graphs (Vygen, 1995). A well-known conjecture by Erdős and Gallai however states that the cardinality of the path partition of a connected graph G which contains the fewest paths is at most $\lceil n/2 \rceil$. As a first step towards proving that conjecture, Lovàsz has shown that a graph G with n vertices, not necessarily connected, can be decomposed in at least $\lfloor n/2 \rfloor$ paths and cycles. More recently, Donald has proven that, for a graph G with u odd-degree vertices and g even-degree vertices, the upper bound for the path number of G is $\lceil u/2 + 3g/4 \rceil$, which is lower than $3n/4$ (Donald, 1980).

In order to properly evaluate the impact of road asymmetry on street networks, two different types of link-disjoint path partitions are here distinguished, namely undirected and directed path partitions. An undirected path partition $\Pi_{E(G)}$ is defined as a family of parts $\pi_1, ..., \pi_n$, which represent pairwise edge-disjoint undirected paths $\pi = P_{E(G)} \langle v_i - v_n \rangle$, and whose union covers $E(G)$. Likewise, a directed path partition $\Pi_{A(G)}$ is a family of parts $\pi_1, ..., \pi_n$ representing pairwise arc-disjoint directed paths $\pi = P_{A(G)} \langle v_1 - v_n \rangle$ whose union covers $A(G)$. In light of these definitions, all street path partitions of undirected planar straight line graphs, corresponding to symmetric road networks, are undirected, while every street path partitioning of mixed planar straight line graphs, related to non-symmetric road networks, will contain both an undirected path partition $\Pi_{E(G)}$ and a directed path partition $\Pi_{A(G)}$.

While some significant breakthroughs have been realized regarding path partitions of planar graphs (Okamura and Seymour, 1981; Schrijver, 1993; Frank, 1982), considerations relative to street path partitions of symmetric and non-symmetric road networks, especially as regards to undirected and directed path partitions, raise specific issues that do not seem to have been addressed so far in both graph theory and road network analysis [7]. Of particular importance to the present research is the problem of specifying the nature of the relation between road direction and street membership, more precisely the impact of the unidirectionality of one-way roads on the formation of street paths and street networks. To address this specific matter, of crucial algorithmic importance, an appeal to supervenience relations seems in order.

3.3.3 SUPERVENIENCE OF DIRECTION ON STREET MEMBERSHIP

Introduced in contemporary philosophy of mind to characterize the relation between mental and physical properties (Davidson, 1980), supervenience relations refer to asymmetric patterns of covariation in object properties.

---

7. A similar project has however been undertaken a few years ago (Courtat et al., 2011b), in which street networks were formally represented as hypergraphs, each street being an hyperedge $HE = \{v_1, ..., v_n\}$ resulting from the union of multiple vertices representing road intersections. While this formalization procedure has many advantages, pragmatic considerations have led the present study to adopt a path partitioning approach. First, the different softwares and extensions used in this research are unable to process hypergraph-related information. But also and more importantly, hypergraph generation would have resulted in too much information loss. First, given that hyperedges are unordered n-tuples of vertices, ordering in road graphs is lost. Moreover and most importantly, all information relative to arc and edges, referring respectively to one-way and two-way road sections, is also lost, as only vertices are represented in hypergraphs. Such information loss in hypergraph generation processes would make the validation of street construction algorithms an extremely difficult task.

> A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties. In slogan form, there cannot be an A-difference without a B-difference (...). Supervenience claims do not merely say that it *just so happens* that there is no A-difference without a B-difference. They say that there *cannot* be one. A-properties supervene on B-properties if and only if a difference in A-properties *requires* a difference in B-properties - or, equivalently, if and only if exact similarity with respect to B-properties *guarantees* exact similarity with respect to A-properties (McLaughlin and Bennett, 2014, p.1-3).

As the use of the italicized words *cannot*, *requires* and *guarantees* in the above quote shows, supervenience relations are necessary ones. However, such necessity can be and has been defined in a number of different ways, resulting in different conceptions and degrees of supervenience, each of which have been proven useful on various theoretical grounds. For the present research, the spatialized version of supervenience proposed by Terence Horgan, called *regional supervenience*, seems well-suited to express the peculiar relationship between direction and topology: by taking space-time regions as individuals instead of objects, regional supervenience expresses "the idea that numerous higher-level properties of individuals are fully determined micro-physically, whether "within the individual itself" or "within a relatively local portion of the universe" (Horgan, 1982, p. 34). McLaughlin reformulated Horgan's principle in two different relations, a weak and a strong one, the latter using possible worlds semantics to extend the scope of supervenience relations to all possible worlds.

> **Weak regional supervenience:**
> A-properties *weakly regionally supervene* on B-properties if and only if for any possible world w and any space-time regions r_1 and r_2 in w, if r_1 and r_2 are B-duplicates in w, then they are A-duplicates in w.

> **Strong regional supervenience:**
> A-properties *strongly regionally supervene* on B-properties if and only if for any possible worlds w_1 and w_2 and any space-time regions r_1 in w_1 and r_2 in w_2, if r_1 is a B-duplicate of r_2 in w_2, then r_1 in w_1 is an A-duplicate of

r_2 in w_2 (McLaughlin and Bennett, 2014, p. 27).

The scope of strong regional supervenience is however too large for the present purposes: since supervenience relationships here only apply to properties of adjacent road sections, appeal to possible worlds semantics and "transworldly" relations seems superfluous, even inadequate. However, by considering a given road network as a possible world and road sections that constitute it as distinct space-time regions, the weaker formulation of regional supervenience allows for a clear and unambiguous definition of the supervenience relation between road direction and street membership:

> **Supervenience of direction on street membership:**
> If road section r_1 is a street-path-duplicate of road section r_2 (if they are part of the same street path), then r_1 is a direction-duplicate of road section r_2 (r_1 and r_2 are both one-way sections pointing in the same direction or two-way road sections).

Given this definition, supervenience of direction on topology means that any difference in direction between two adjacent roads entails and even requires a difference in street membership. Hence, two one-way road sections that point in opposite directions or two road sections that are of different direction types, one being one-way and the other two-way, are also and necessarily part of different street paths.

In terms of both structure and dynamics, this condition can be easily explained, as a road direction change in the network necessarily entails a change in structure, navigability and use. Indeed, two adjacent road segments do not "connect" in the same way when the direction type of one of them is modified: if two adjacent two-way road sections are made direction-incompatible by converting one of them into a one-way, an important breach in continuity occurs, since the accessibility relation between both sections is no longer symmetrical, and an important detour must be made from one road section in order to reach the other one. Given that the concept of street is fundamentally based on continuity, such a change necessarily implies a consequent change in street structure.

It must be noted here that the relation of covariation implied in this supervenience of direction on street membership is ontologically innocent, in that it doesn't imply any ontological priority or dependence. "Supervenience claims, by themselves, do nothing more than state that certain patterns of property (or fact) variation hold. They are silent about why those patterns hold, and about the precise nature of the dependency involved" (McLaughlin and Bennett, 2014, p.20).

> Often, when someones asserts that A supervenes on B, she also wants to say that A properties *ontologically depend* upon B properties, regardless of whether or not they are entailed by the B-properties, or count as further ontological commitment. However, even this goes this beyond the minimum required for supervenience. Supervenience is not a relation of ontological priority; the supervenience of A on B does not guarantee that B-properties are ontologically prior to A-properties (McLaughlin and Bennett, 2014, p. 17).

The case of necessary and impossible properties, that is, properties that respectively every and no entity can have, might help distinguish supervenience claims from ontological priority ones. For example, every road section has the property *being a one-way or not a one-way*, while no road section can have the property *being both a one-way and not a one-way*. Now, given that no two things can differ relatively to necessary or impossible properties, it follows that no two things can differ with respect to *being both a one-way and not a one-way* or *being a one-way or not a one-way* without also differing with respect to any other property, for instance *being a two-way*, *being red-haired* and so on. But such considerations by no means imply that *being both a one-way and not a one-way* or *being a one-way or not a one-way* ontologically depends on any other property.

Regardless of these ontological considerations, supervenience of direction on street membership has a huge impact on the path partition of road networks: any street path can include one-way road section or two-way road sections, but never both. This supervenience relation as well as its effect on the different street types and global street structure will be better explained in the following section, in which a detailed description of the different street partition generation algorithms is given.

3.3.4 STREET PATH PARTITIONS OF ROAD NETWORKS

Given the supervenience of road direction on street membership explained in section 3.3.3, one-way and two-way road sections necessarily form distinct streets. Also, one-ways road sections that are part of the same street are necessarily direction-compatible, that is, they must all point in the same direction. In this sense and in accordance with the definitions given in section 3.3.1, all streets are either directed paths or undirected paths; correspondingly, all path partitions of undirected planar straight line graphs have only undirected paths as elements, while any path partition of a mixed planar straight line graph is made of both undirected and directed paths. Algorithm **??** describes the procedure for the different street partitionings of all planar straight line graphs generated.

For each link l in G (line 2), if that arc or edge is not already part of a path (line 10), then create a new path π_i containing that link as element (line 11). Then, the algorithm builds the path π_i by trying to expand it both backward and forward: if the link extremity corresponds to the tail or first end of *link* (lines 5 and 8), backward path expansion is executed; otherwise (lines 6 and 9), forward path expansion is executed. Three different types of path expansions are distinguished: if the street type is segmental (line 12), then the procedure Extend-Segmental-Path is launched for both ends of *link* (line 13); if the street type is odonymic (line 14), then the procedure Extend-Odonymic-Path is called for both directions (line 15); finally, if the street type is angular (line 16), Extend-Angular-Path is executed, again on both extremities of *link* (line17). In the following paragraphs, path construction procedures relative to each street type (Extend-Segmental-Path, Extend-Odonymic-Path, Extend-Angular-Path) are described, accompanied by simple road network examples that highlight the structural peculiarities and impact of each street type.

Input: *PSLG*, StreetType, FitnessType, Threshold
Output: $\Pi(PSLG)$

begin
  i=0;
  foreach $x \mid x \in (A(PSLG) \cup E(PSLG))$ **do**
    link=x;
    if link $\in A(PSLG)$ **then**
      e1=tail(link);
      e2=head(link);
    else
      e1=$\{e \mid (\mathscr{R}(\text{link}) = e) \wedge (\mathbb{P}(\mathscr{R}(\text{link}) = e) = 1/2)$;
      e2=$\{e \mid e \in \text{link} \wedge e \neq e1\}$;
    if $\neg\exists\pi \in \Pi(PSLG)(x \in \pi)$ **then**
      $\pi_i = \langle x \rangle$;
      if *StreetType*= *"segmental"* **then**
        Extend-Segmental-Path(e1,link,π_i,backward)
        Extend-Segmental-Path(e2,link,π_i,forward)
      if *StreetType*= *"odonymic"* **then**
        Extend-Odonymic-Path(e1,link,π_i,backward)
        Extend-Odonymic-Path(e2,link,π_i,last,forward)
      if *StreetType*= *"angular"* **then**
        Extend-Angular-Path(e1,link,π_i,backward,FitnessType,Threshold)
        Extend-Angular-Path(e2,link,π_i,forward,FitnessType,Threshold)
      $i = i + 1$;

Algorithm 2: Path Partition Generation Algorithm

3.3.4.1 SEGMENTAL STREETS

Segmental streets represent the easiest street type to understand and implement as well as the most common, as "most urban analyzes use street segments as the building block of the street pattern" (Tomko et al., 2008, p. 43-44). In the same way that a line segment corresponds to the part of a line bounded by two endpoints, a road segment represents the portion of a road comprised between two intersections or between an intersection and a dead end. Thus, in segmental street partitions, all links of degree 2 comprised between pairs of vertices corresponding to adjacent intersections or adjacent intersection/dead end pairs in the road network are contained in a single street path.

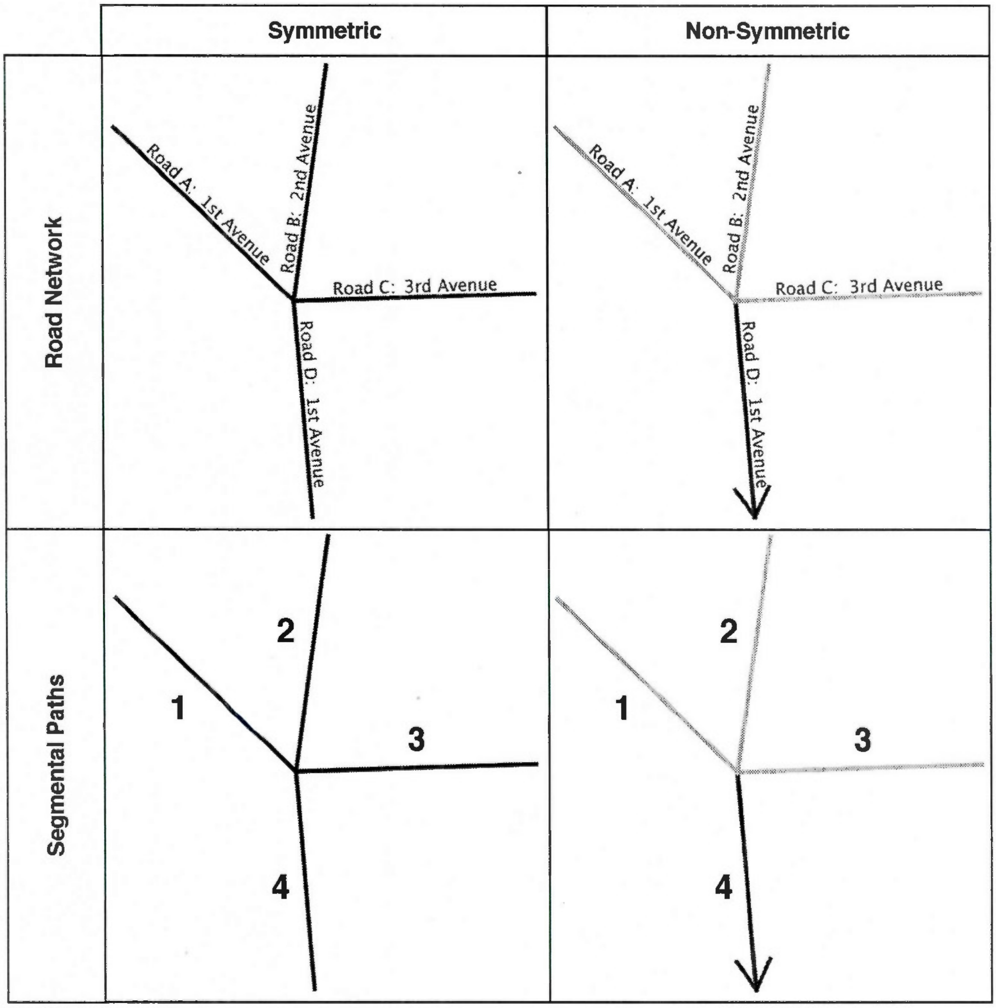
Such condition allows for the exclusion of all vertices that function as "additional points used (...) to sample curved streets into straight segments" (Courtat et al., 2011a, p. 317).

Here as in other cases, street membership depends on road direction: in the case of downtown London, many two-way roads become one-ways before connecting with major roads in order to allow access from one side of the road only. This structural characteristic of the London road network infrastructure cannot be overlooked, as it has a non-negligible impact on road-use and overall urban movement. The procedure for the construction of segmental street paths, Extend-Segmental-Path, is as follows: given the set of all arcs or edges sharing node *end* with arc or edge *link* (line 2), if that set contains only one arc or edge (line 3) which is direction-compatible with *link* (both are edges or are arcs of distinct head and tail) and not included in any existing path (line 5), then *link*2 is added either before (line 8) or after (line 10) *link* in the current path, depending on the initial direction (line 7). If the segmental street path is duly extended, Extend-Segmental-Path is executed again on *link*2 and its other end (the one not included in *link*), with the same path and direction as other inputs (line 11). That procedure is carried on until no other links satisfy the segmental street path extension requirements. Used in conjunction with the above described algorithm, Extend-Segmental-Path allows for the generation of a set of segmental street paths covering the road graph integrally.

1 **begin**
2   Linkneighbours=$\{l \mid l \in \{A(PSLG) \cup E(PSLG)\} \wedge end \in l \wedge l \neq link\}$;
3   **if** $|Linkneighbours| = 1$ **then**
4     $link2 = l \mid l \in Linkneighbours$;
5   **if** $((link, link2 \in A(PSLG) \wedge ((tail(link) = head(link2)) \vee (head(link) = tail(link2))) \vee link, link2 \in E(PSLG))$ **then**
6   otherEnd=$\{e \mid e \in link2 \wedge e \notin link\}$;
7   **if** *direction=backward* **then**
8         $path = \langle link2, path \rangle$
9   **else**
10         $path = \langle path, link2 \rangle$
11   Extend-Segmental-Path(otherEnd,link2,path,direction);

Procedure Extend-Segmental-Path(end,link,path,direction)

Figure 8 – Segmental Path Example



As regards to the road network example shown in Figure 8, segmental paths of symmetric and non-symmetric road networks do not present any differences, but this does not imply however that symmetric and non-symmetric segmental street networks are identical. On the contrary, it must be remembered that in the three London neighbourhoods under study, many changes in direction type occur between intersections; given the supervenience of direction on street membership, such differences in direction implies that non-symmetric segmental street networks will contain more streets than their symmetric counterpart. Moreover, the fact that the only one-way road section shown in the road example of Figure 8 is directed away from the three other road sections has important implications as regards to accessibility: while the one-way section can be reached from the three two-way road sections, the reverse is not true, and this accessibility asymmetry has important topological implications. More will be said on that matter in section 3.4.1.

3.3.4.2 ODONYMIC STREETS

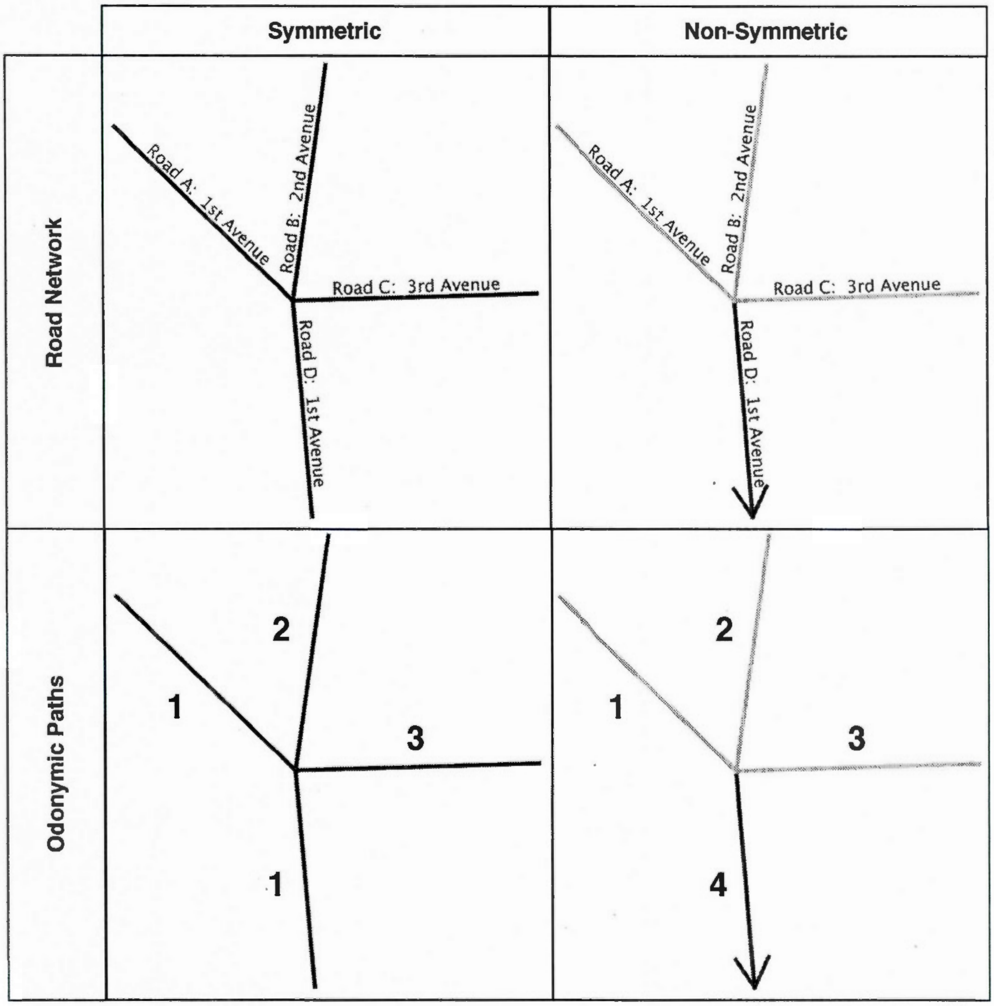
Odonymic streets are made of adjacent road sections that bear the same odonym or street name. Construction of such street paths is carried out through the Extend-Odonymic-Path procedure.

1 **begin**
2   Linkneighbours=$\{l \mid l \in \{A(PSLG) \cup E(PSLG)\} \wedge end \in l \wedge l \neq link \wedge \neg \exists \pi \in \Pi(l \in \pi)\}$;
3   link2=$l \mid l \in Linkneighbours \wedge Name(l) = Name(link)$;
4   **if** $((link, link2 \in A(PSLG) \wedge ((tail(link) = head(link2)) \vee (head(link) = tail(link2))) \vee link, link2 \in E(PSLG))$ **then**
5   otherEnd=$\{e \mid e \in link2 \wedge e \notin link\}$;
6   **if** *direction=backward* **then**
7       $path = \langle link2, path \rangle$
8   **else**
9       $path = \langle path, link2 \rangle$
10   Extend-Odonymic-Path(otherEnd,link2,path,direction);

Procedure Extend-Odonymic-Path(end,link,path,direction)

This procedure, fairly similar to Extend-Segmental-Path, can be read as follows. Given the set of all arcs or edges sharing node *end* with arc or edge *link* (line 2), let *link2* be the arc or edge contained in that set whose (odonymic) label is the same as the label of *link* (line 3). Now, if *link2* is direction-compatible with *link* (both are edges or arcs with distinct head and tail) and not included in any existing path (line 4), then *link2* is added either before (line 7) or after (line 9) *link* in the current path, depending on the initial direction (line 6). If the odonymic street path is duly extended, Extend-Odonymic-Path is executed again on *link2* and its other end (the one not included in *link*), with the same path and direction as other inputs (line 10). That procedure is carried on until no other links satisfy the segmental street path extension requirements. Used in conjunction with the above described algorithm, Extend-Odonymic-Path allows for the generation of a set of odonymic street paths covering the road graph integrally.

Figure 9 – Odonymic Path Example



As can be seen in the road example of Figure 9, road sections A and D have the same odonym (1st avenue). In the symmetric road network, this results in the creation of a odonymic path including both A and B (path number 1), while road segments B and C have distinct odonyms (2nd and 3rd avenue respectively) and are thus included in distinct street paths. If direction type is taken into account, however, a different network results: despite being adjacent and bearing the same odonym, road segments A and D are direction-incompatible, the former being a two-way road section and the latter a one-way type. Thus, given that direction supervenes on street membership, these segments cannot be included in the same street path.

3.3.4.3 ANGULAR STREETS

More complex than its segmental and odonymic counterparts, the construction of angular street paths depends on two different parameters, angular fitness type and angular threshold, both of which are based on considerations relative to deflection angle values. The deflection angle, referred to here as $\gamma(l_1, l_2)$, is here defined as the angle between a given line l_2 and the prolongation of the preceding line l_1. In short, it designates "the amount of deviation that has occurred from the direction of travel" (Field, 2012, p.97). Counterclockwise deviation is referred to as negative or left deflection, while clockwise deflection is designated by positive or right deflection. In the present research, only the absolute (positive) value of deflection angles is considered, which means that the deviation of any line from the prolongation of a preceding line is always between 0^o and 180^o. A deflection angle of 0^o implies the absence of any deviation and a deflection angle of 180^o amounts to a total direction reversal. Unsurprisingly, the latter, extreme, case is absent from the road networks investigated.

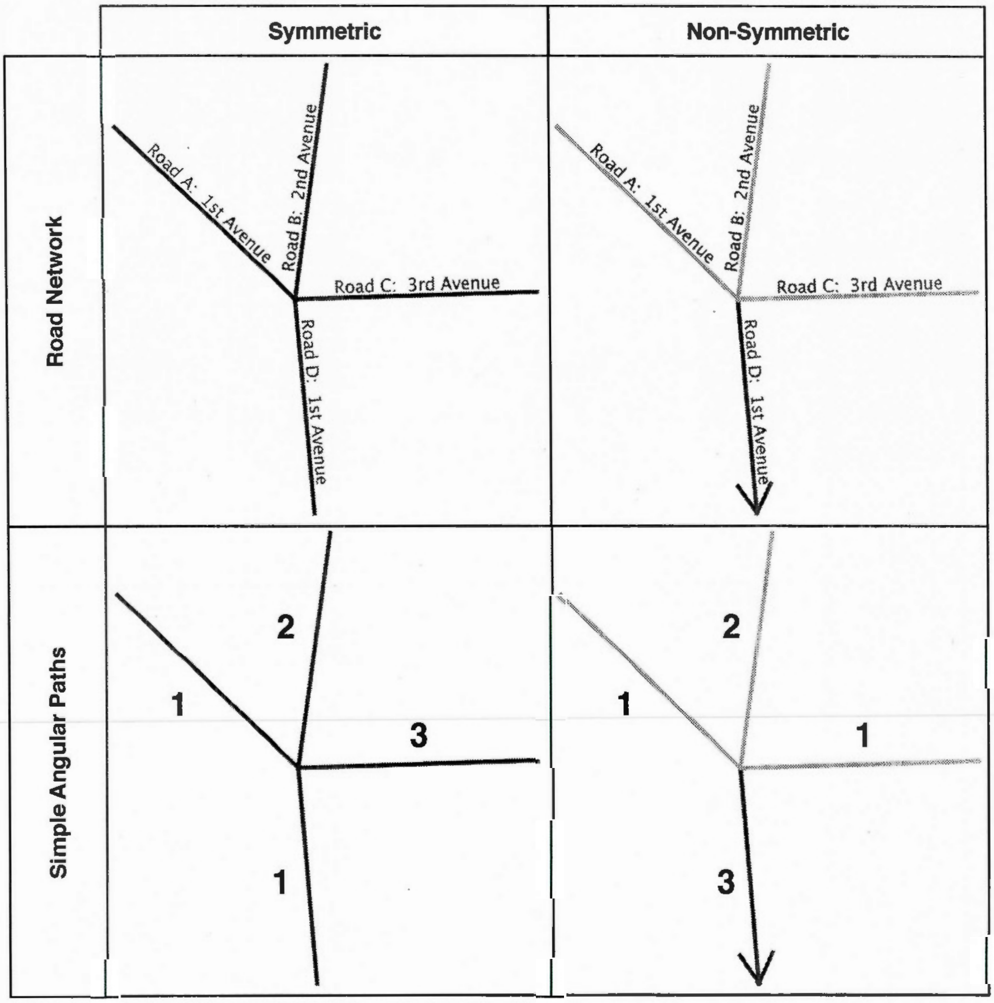
Out of the two angular parameters, threshold is the easiest to understand and implement: it corresponds to the maximum possible deflection angle value between two adjacent arcs or edges of any given path; in other words, if the deflection angle formed by two adjacent links is above the angular threshold, both must necessarily belong to different angular street paths. Inversely, all links adjacent to another link and forming a deflection angle smaller than the fixed threshold value are considered valid angular candidates. For example, in one of the first studies using angularity-based street partitions, adjacent road sections were considered as potential street extensions only if their angle was less or equal than 45^o Figueiredo and Amorim (2005).

As for angular fitness type, it relates to different deflection-angle-based criteria for the selection of potential path extensions from a set of valid angular candidates (Jiang, 2009a). The first and simplest angular fitness criterion is based on the best angular fit for a given road section. However, since the use of this criterion generates street networks that are highly dependent on input ordering, more sophisticated angularity-based merging conditions have been devised and implemented for street-based models. Thus, in Porta et al. (2006a), two axes are merged at an intersection if they make the largest convex angle between all angles at the intersection. Finally, in Jiang (2009a), different segment merging principles (every-best-fit, self-best-fit, self-fit) were implemented and tested on the basis of all angles formed by incident segments.

In the present research, three different angular fitness types are considered and implemented, namely 1) simple fitness, 2) relative fitness and 3) mutual fitness. According to simple angular fitness, the best angular fit for a given link l_1 and end e is the link l_2 incident with e and forming the best valid angular candidate for l_1, that is, the link forming the lowest valid (under angular threshold value) deflection angle with l_1 and e. As its name implies, simple angular fitness is the easier to understand and implement of all three. However, simple angular fitness generates graphs that are strongly determined by the ordering of the data given as input: modifying the sequence of links given as input will more most likely generate a different output. For example, the deflection $\angle l_1, e, l_2$ may be the smallest of all deflection angles including l_1, but given that there may well exist a link l_3 such that $\angle l_3, e, l_2 < \angle l_1, e, l_2$, the street graph returned as output will ultimately depend on whichever link, either l_1 or l_3, comes first as input.

Using the same road example as before, Figure 10 shows how simple angular paths are constructed. Assuming that road sections are processed in alphabetical order, simple angular paths in symmetric context works the following way: since the $\angle AB$ deflection angle is greater than any possible angular threshold and given that the $\angle AD$ deflection angle is slightly smaller than $\angle AC$, road sections A and D are included in the same simple angular path (path number 1); as for road sections B and C, their deflection angle $\angle BC$ is above threshold, thus resulting in distinct street paths (paths 2 and 3). If road direction in taken into account, however, a rather different street network results: since A and D are direction-incompatible, the best angular fit for A would be C (path 1), while road sections B and D would be part of different paths (paths number 2 and 3). As said earlier, this simple angular path construction procedure however depends

Figure 10 – Simple Angular Path Example

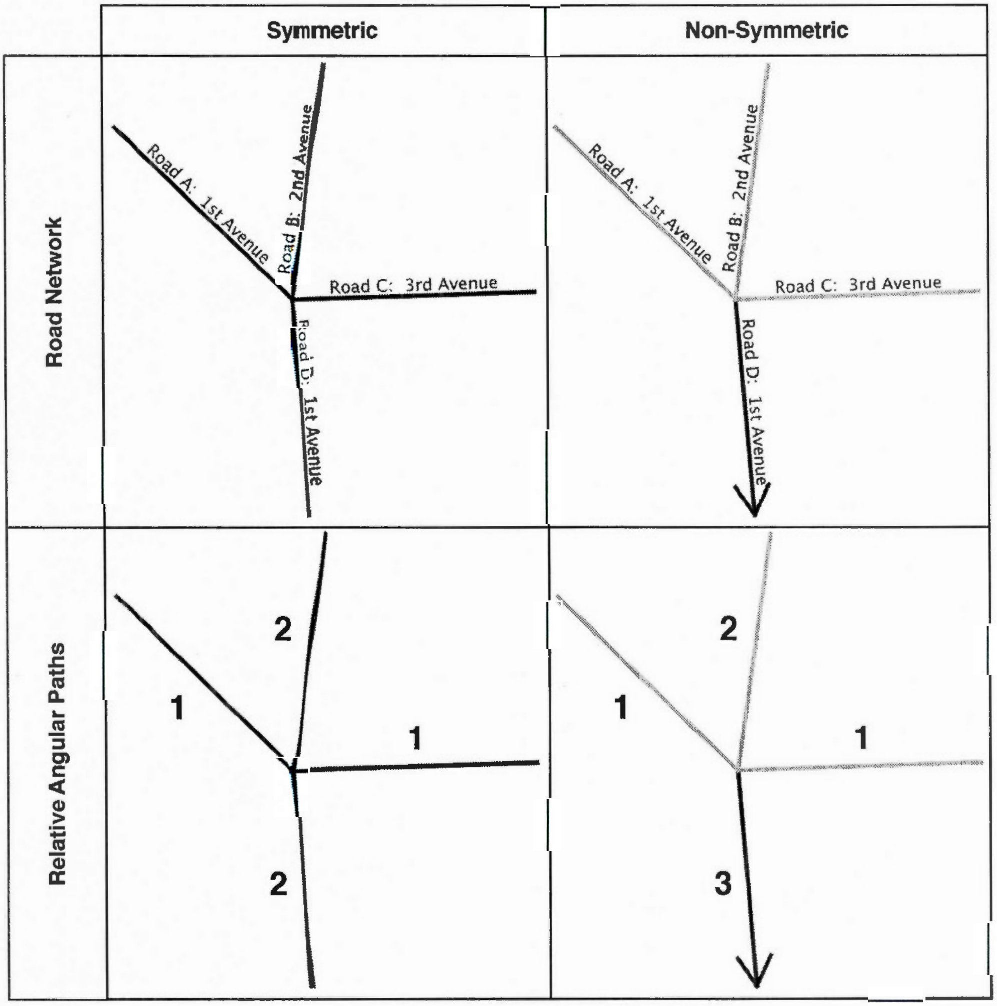


on input ordering: if the same same road network is processed in reverse order (from D to A), the best angular fit for D would be road section B, while the deflection angle between C and A could be low enough to allow for both road sections to be included in the same simple angular street path.

As for relative angular fitness, it selects, from all links adjacent to l_1, incident with e and having l_1 as best angular fit, the one which forms the lowest deflection angle with l_1. This angular fitness type is named "relative" due to the fact that it takes into account the angular context of all valid links adjacent to l_1 and incident with e, not only that of

l_1.

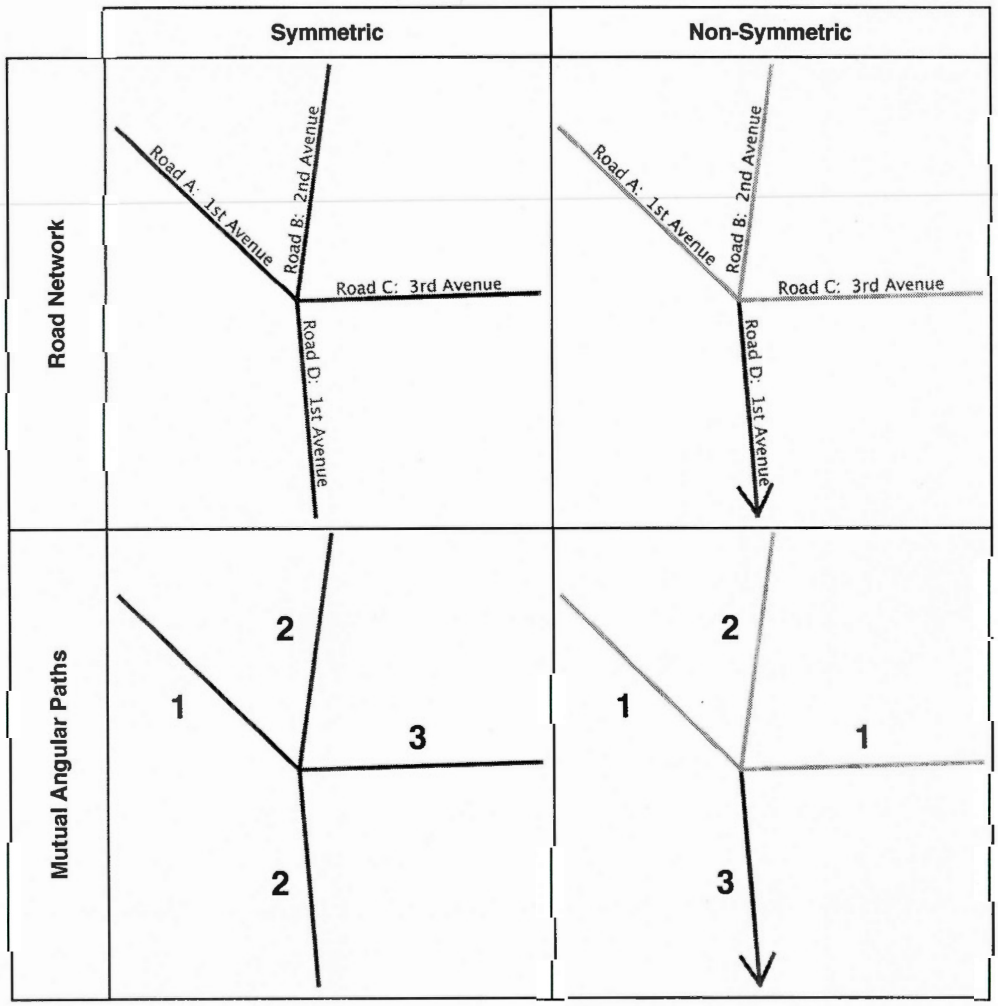
Figure 11 – Relative Angular Path Example



Returning to the previous road network example, Figure 11 shows how the relative angular path construction procedure works. Since C is the only road section having A as best angular fit (the best angular fit of D is B), A and C are included in the same path (path number 1), while B represent the best fit of D, which results in their common path inclusion (path number 2). It is easy to show here how this procedure is independent of input ordering: if roads are processed in reverse order, both A and B have D as best fit; however, since the deflection angle $\angle BD$ is smaller than $\angle AD$, B and D are joined in the same path as before (path numbering would be different, however); as for C, only A has it as best fit, which would also result in the creation of a path identical to the one generated in the previous situation. As regards to road direction, it is interesting to note that, despite generating different path partitions in symmetric context, both simple and relative angular procedures produce identical path structures when road direction is taken into account.

Finally, mutual angular fitness combines both simple and relative angular fitness types: a given link l_1 merges with a link l_2 adjacent with l_1 and incident with e if the deflection angle $\angle l_1, e, l_2$ is the lowest of all deflection angles involving either l_1, e or l_2. In other words, l_2 is mutually and angularly fit for l_1 if the best angular fit of l_1 is l_2 and the best angular fit of l_2 is l_1.

Figure 12 – Mutual Angular Path Example



The road network given previously as example suffices to show how differences in angular fitness - in the present case, choice of mutual angular fitness - results in different partitioning scenarios. In the symmetric case, road section A has D for best fit; however, that relation is not mutual, as the best fit of D is B, which results in A being left unmerged (path number 1). Now, given that B and D represent mutual best angular fits, a common path is created for them (path number 2), thus leaving C alone (path number 3). Here again, path partitioning is independent of input ordering: D and B are mutual best fits and are thus joined together, while A and C are not mutual best fits and thus put into different paths. In non-symmetric context, a partition identical to those generated using the other angular fitness types is obtained: D being direction-incompatible with the other road sections (path number 3), A and C represent best mutual angular fits and are thus merged (path number 1), while B is left alone (path number 2). Thus, despite generating distinct partitions in a small symmetric network, the different angular fitness types produce identical configurations in non-symmetric context, a fact which certainly highlights the impact of road asymmetry on street network structure.

Following proper explanation and exemplification of the different angular fitness types, the procedure used in the construction of angular street paths, **??**, can be described as follows:

For any link *link* and end *end* given as inputs, if *link* is an arc (line 2), the set of potential angular street path extensions for *link* includes all arcs either incoming to (line 4) or outgoing from (line 6) *end* and forming with *link* a deflection angle lower than or equal to the angular threshold value. If, on the other hand, *link* is an edge (line 8), the same potential angular street path extension set is made of all links adjacent to *link* and incident with e (line 8). Following this, if the angular fitness type is simple (line 9), let *link*2 be the member of the set whose deflection angle with *link* is the smallest (line 10). Likewise, if the angular fitness type is relative, select from the above set all links whose simple angular fit is *link* (line 12) and let l_2 be the member of that smaller set whose deflection angle with *link* is the smallest (line 13). Thirdly, if the angular fitness type given as input is mutual (line 14), let *link*2 be the member of the potential angular street path extension set whose simple angular fit is *link* and who is

1 **begin**
2 **if** $link \in A(PSLG)$ **then**
3 **if** $direction = backward$ **then**
4 LinkHood=$\{x \mid x \in A(PSLG) \wedge head(x) = tail(end) \wedge \gamma(link,x) \leq Threshold \wedge \neg\exists\pi \in \Pi(x \in \pi)\}$;
5 **else**
6 LinkHood=$\{x \mid x \in A(PSLG) \wedge tail(x) = head(end) \wedge \gamma(link,x) \leq Threshold \wedge \neg\exists\pi \in \Pi(x \in \pi)\}$;
7 **else**
8 LinkHood=$\{x \mid x \in E(PSLG) \wedge end \in x \wedge x \neq link \wedge \gamma(link,x) \leq Threshold\}$;
9 **if** *FitnessType*=*"simple"* **then**
10 link2=$\{x \mid (x \in LinkHood) \wedge \forall y((y \in LinkHood) \wedge (y \neq x) \rightarrow (\gamma(link,x) < \gamma(link,y)))\}$;
11 **if** *FitnessType*=*"relative"* **then**
12 PotentialExtensions=$\{x \mid (x \in LinkHood) \wedge \forall y((y \in LinkHood) \wedge (y \neq x) \rightarrow (\gamma(x,link) < \gamma(x,y)))\}$;
13 link2=$\{x \mid (x \in PotentialExtensions) \wedge \forall y((y \in PotentialExtensions) \wedge (y \neq x) \rightarrow (\gamma(x,link) < \gamma(y,link)))\}$;
14 **if** *FitnessType*=*"mutual"* **then**
15 link2= $\{x \mid (x \in LinkHood) \wedge \forall y(((y \in LinkHood) \wedge (y \neq x)) \rightarrow ((\gamma(link,x) < \gamma(link,y)) \wedge \neg\exists z((z \in LinkHood) \wedge (z \neq x) \wedge (\gamma(x,z) < \gamma(x,link)))))\}$;
16 **if** $link2 \neq \emptyset$ **then**
17 otherEnd=$\{e \mid e \in link2 \wedge e \notin link\}$;
18 **if** *direction=backward* **then**
19 $path = \langle link2, path \rangle$
20 **else**
21 $path = \langle path, link2 \rangle$
22 Extend-Angular-Path(otherEnd,link2,path,direction, fitnessType, Threshold);

Procedure Extend-Angular-Path(end,link,path,direction,FitnessType,Threshold)

also and reciprocally the simple angular fit of *link* (line 15). Following this series of conditionals, if *link*2 does not correspond to the empty set and isn't part of any existing path (line 16), then *link*2 is added either before (19) or after (21) *link* in the current path, depending on the initial direction (line 18). If the street path is duly extended, Extend-Angular-Path is executed again on *link*2 and its other end (the one not included in *link*), with the same path and direction as other inputs (line 2). That procedure is carried on until no other links satisfy the angular street path extension requirements. Such procedure allows for the generation of a set of angular street paths covering the road graph integrally.

Finally, given that there is no definitive and objective value that can be used as standard angular threshold, different angular thresholds have been used in the generation of street paths and partitions: for each road graph (Barnsbury symmetric, Barnsbury non-symmetric, Clerkenwell symmetric, Clerkenwell non-symmetric, Kensington symmetric, Kensington non-symmetric) and angular fitness type (best, relative, mutual), different street paths and partitions were generated for each of these angular threshold values: 20^0, 30^0, 40^0, 50^0, 60^0, 70^0. Given that one street partition is generated for each possible parametrization setting, a total of 120 street path partitions have been generated in this research.

3.4 DIRECTED GRAPH GENERATION PROCEDURES

Following the construction of all 120 street partitions and in order to obtain the ω scores for each street network, three different directed graph types have to be generated. First, street networks, defined here as intersection graphs of path partitions, are build by having one vertex for each path and, for each pair of paths, an arc from one path to another if the latter can be reached directly from the former. Following generation of these street networks and in conformity with the Watts-Strogatz model, average clustering coefficient and characteristic path length values are calculated for each of them. Secondly, random graphs isosequential to each different street network are generated using a "link-swapping" procedure described below, after which the characteristic path length of each random graph is recorded in order to compare it to the characteristic path length the corresponding street networks. Finally, isosequential highly-clustered graphs are generated by means of a second "link-swapping" procedure and their average clustering coefficient is recorded for further comparison with results obtained for the corresponding street networks. It must be noted here that the isosequential random graph and isosequential highly-clustered graph generation procedures are heuristics, since they do generate, for any given in-degree and out-degree sequences, optimal graphs as regards to characteristic path length minimization and average clustering coefficient maximization respectively. Given this, both link-swapping procedures are executed 10 times, after which the mean characteristic path length and mean average clustering coefficient of all isosequential random graphs and isosequential highly-clustered graphs are respectively kept for ω score calculation.

All these graphs are all made directed in order to properly express non-symmetry, in other words the fact that at least one adjacency relation is asymmetric. In non-symmetric context, asymmetric adjacency relations necessarily result in differences in the in-degree and out-degree sequences, which means that all random graphs and highly-clustered graphs generated must also be directed in order to be isosequential to their corresponding street graph.

Moreover and as before, all directed graphs are geometric, that is, embedded in the plane. This may indeed appear surprising, given that calculation of average clustering coefficient, characteristic path length and ω scores only required topological information. However, given that intersection graphs, isosequential random graphs and isosequential highly-clustered graphs are still embedded in NetLogo space, which is a cartesian plane, geometric coordinates are still needed for vertices. It is also important to stress that this metric embedding allows for the definition of a distance function, which will prove essential to the highly-clustered graph generation algorithm described below.

Finally and most importantly, modified average clustering coefficient and characteristic path length definitions must be provided, as the ones given by Watts and Strogatz only apply to undirected graphs. As in the Watts-Strogatz model, the clustering coefficient of a vertex measures the "cliquishness" of its neighbourhood, measured as the probability that all links incident with it are adjacent to each other, while the average clustering coefficient refers to the same probability, averaged for all vertices of the graph. However, given that in directed graphs, any pair of vertices can be connected twice, with one arc directed in one side and another one directed in the other, there can be up to twice as many arcs as edges for the same number of vertices. In order to account for this fact, clustering coefficient for directed graphs is defined in the following way:

(4) $$C_G(v) = \frac{||N_v||}{|N_v| \times (|N_v| - 1)},$$

where v refers to any given vertex $v \in V(G)$. As for N_v, it denotes the neighbourhood of v, $N_v^+ \cup N_v^-$, that is, the subset of vertices $\in V(G)$ which are adjacent to v, whether as head of an arc directed from v or a tail of an arc directed to v. In light of this, $|N_v|$ refers to the number of vertices of the neighbourhood and $||N_v||$ to the total number of arcs between them. When averaged over all vertices of G, this clustering coefficient allows measurement of the global "cliquishness" of the network, here defined as:

(5) $$\langle C \rangle_G = \frac{1}{|V(G)|} \sum_{v \in V(G)} C_G(v).$$

As for the characteristic path length, is it based on the shortest path length between two distinct vertices $v_1, v_2 \in V(G)$, denoted as $\ell_G(v_1, v_2)$ and referring to the length of the shortest of all $v_1 - v_2$ paths in G:

(6) $$\ell_G(v_1, v_2) = \min ||P_G(v_1, v_2)||.$$

It must be reminded here that the path length between any two vertices of a directed graph is often different from the corresponding path length of the corresponding underlying graph. Indeed, while edges can be crossed in both ways, that is, from any edge end to the other, all directed paths are defined in this research as forward paths, according to which an arc can only be crossed from its tail to its head. Given this, the eccentricity $ecc(v)$ of a vertex v corresponds to the maximal length of any path in G having v as initial vertex. Eccentricity is related to two structural indicators closely related to characteristic path length, that is the radius and diameter of a graph: the radius of G, r_G, refers to the minimum eccentricity among all vertices $v \in V(G)$; as for diameter of G, d_G , it is equal to the maximum eccentricity for all vertices in $V(G)$. Formally,

$$
\begin{aligned}
ecc(v_1) &= \max \{\ell_G(v_1, v_2) \mid v_2 \in V(G) \wedge v_2 \neq v_1\} \\
r_G &= \min \{ecc(v) \mid v \in V(G)\} \\
d_G &= \max \{ecc(v) \mid v \in V(G)\}.
\end{aligned}
$$

Given these definitions, the characteristic path length $\langle \ell \rangle_G$ of a graph G is the average of the shortest paths from all vertices in $V(G)$ to all other vertices of the same set. In short, it measures the average distance from every vertex to every other vertex in the graph:

$$
\langle \ell \rangle_G = \frac{1}{|V(G)| \times (|V(G)| - 1)} \sum_{v_1 \in V}^{|G|} \sum_{\substack{v_2 \in V \\ v_2 \neq v_1}}^{|G|} \ell_G(v_1, v_2).
$$

For unweighted graphs such as those considered in this research, this characteristic path length is equivalent to the average geodesic distance. Of course, characteristic path length is only effective with connected graphs, in which there exists a path from each vertex to every other vertex; in the case of an isolated vertex, its distance to all other graph vertices would be infinite, thus rendering the measure meaningless.

Given both definitions and the above considerations, the following subsections introduce, describe and justify the procedures designed for intersection graph, isosequential random graph and isosequential highly-clustered graph generation.

3.4.1 INTERSECTION GRAPHS (STREET NETWORKS)

Intersection graphs are usually defined as graphs that represent intersection patterns in a family of sets. In general, intersection graphs are undirected: for any family F, an intersection graph has one vertex for each family member and edges between family members if their intersection set is nonempty, that is, if they have common members:

> Let $\mathscr{F}$ be a family of sets. The *intersection graph of* $\mathscr{F}$, denoted $\Omega(\mathscr{F})$, is the graph having F as vertex set with S_i adjacent to S_j if and only if $i \neq j$ and $S_i \cap S_j \neq \emptyset$. A graph is an *intersection graph* if there exists a family $\mathscr{F}$ such that $G \cong \Omega(F)$, where we typically display this isomorphism by writing $V(G) = \{v_1, ..., v_n\}$ with each v_1 corresponding to S_i; thus $v_i v_j \in E(G)$ if and only if $S_i \cap S_j \neq \emptyset$. When $G \cong \Omega(\mathscr{F})$, $\mathscr{F}$ is then called a *set representation* of G (McKee and McMorris, 1999, 1-2).

Defined as such, intersection graphs constitute very general structures. In fact, any undirected graph may be represented as an intersection graph, by forming for each vertex set $v_i \in V(G)$ a link-set S_i consisting of all edges incident to v_i, such that any intersection between two link sets S_i and S_j is nonempty if and only if there is an edge between v_i and v_j (Szpilrajn-Marczewski, 1945; Culik, 1964; Erdős et al., 1966). Moreover, the generality of intersection graphs is such that many different graphs types can be described as intersection graphs of specific types of set families. For example, interval graphs can be defined as intersection graphs representing intersection patterns in families of intervals on the real line and circle graphs as intersection graphs of a set of chords of a circle. However, line graphs probably represent the best known and studied type of intersection graphs. First used in Whitney (1932) and Krausz (1943), line graphs received their current name in an article by Harary and Norman Harary and Norman (1960) [8]. Formally, for any given graph G, the line graph $L(G)$ is the intersection graph of edges in $E(G)$, such that each vertex in $L(G)$ corresponds to an edge in $E(G)$ and two vertices in $L(G)$ are joined by an edge if their corresponding edges in G share a common vertex (McKee and McMorris, 1999).

Intersection graphs are no strangers to transportation infrastructure analysis, as line graphs have already been used to model turn restrictions in road networks (Winter, 2002; Caldwell, 1961). In the case of the present research, however, both line graphs and intersection graphs are unable to adequately express intersection patterns in street path partitions. First of all, since street path partitions contain ordered sets of arcs and edges of cardinality higher than or equal to 2, they cannot by definition constitute proper set representations for line graphs. Moreover, given the non-symmetrical nature of adjacency relations in non-symmetric street networks, intersection patterns among directed and undirected street paths cannot be properly formalized by edges, but only using arcs. In other words, only directed intersection graphs can properly represent non-symmetrical relations among the directed and undirected street path partitions.

First introduced by Beineke and Zamfirescu (Beineke and Zamfirescu, 1982) and Maehara (Maehara, 1984), directed intersection digraphs were later generalized by Sen, Das, Roy and West in a series of articles focusing on interval digraphs (Das et al., 1989; Das and Sen, 1993; Sen et al., 1995; West, 1998):

> Let $\{\langle S_v, T_v\rangle\}$ be a collection of ordered pairs of sets indexed by a set V; we call S_i the *source set* and *the sink set* or v. The *intersection digraph* of

8. However, line graphs have been called a number of different ways over the years; to name but a few: congruent graphs (Whitney, 1932), derivative graphs (Sabidussi, 1961), dual (Anez et al., 1996) or edge-to-vertex dual graphs (Seshu and Reed, 1961), interchange graphs (Ore, 1962), covering graphs (Kasteleyn, 1967), derived graphs (Beineke, 1968), adjunct graphs (Menon, 1966; Berge, 2001). This denominational diversity may have helped underestimate the importance of line graphs in the graph-theoretical and mathematical literature in general

> this collection is the digraph with vertex set V having an [arc] from i to j if and only if $S_i \cap T_i \neq \emptyset$ (West, 1998, p.287).

In the case of street path partitions of G, which are ordered sets of arcs or edges, directed intersection graphs can thus be generated by adding one vertex for each street path and by creating an arc from a vertex v_1 to another vertex v_2 not only if the path corresponding to v_1 has a vertex in common with the path represented by v_2, but also if certain conditions relating to the asymmetry of directed paths are met. Thus, in order to generate directed intersection graphs that are sensitive to the structural constraints relative to road direction, the following algorithm has been devised.

Input: $\Pi(PSLG)$
Output: $I(\Pi(PSLG))$
begin
  $i = 0;$
  while $i <\mid \Pi \mid$ **do**
    $thisX = 2{-}1(\max_{v \subseteq \pi_i} first(v_i) + \min_{v \in \pi_i} first(v_i));$
    $thisY = 2{-}1(\max_{v \subseteq \pi_i} last(v_i) + \min_{v \in \pi_i} last(v_i));$
    $V(I) = \{V(I) \cup v_i = \langle thisX, thisY \rangle\};$
    $i = i + 1;$
  $i = 0;$
  while $i <\mid \Pi \mid$ **do**
    j=0 **while** $j <\mid \bigcup \Pi \mid$ **do**
      if $i \neq j$ **then**
        if $\exists a(a \in p_i)$ **then**
          FirstPoint=$\{x \mid (\exists a(a \in p_i \wedge tail(a) = x) \wedge \neg \exists b(b \in p_i \wedge head(b) = x))\};$
        if $\exists a(a \in p_j)$ **then**
          LastPoint=$\{x \mid (\exists a(a \in p_i \wedge head(a) = x) \wedge \neg \exists b(b \in p_i \wedge tail(b) = x))\};$
        if $(\exists x \exists a \exists b((x \in a) \wedge (a \in \pi_i) \wedge (x \in b) \wedge (b \in \pi_j)) \wedge (x \neq (FirstPoint \vee LastPoint)))$ **then**
          $(A(I) = \{A(I) \cup \langle v_i, v_j \rangle\});$
      $j = j + 1$
    $i = i + 1;$

Algorithm 3: Intersection Graph Generation Algorithm

For each path $\pi_i \in \Pi(G)$ (line 3), a vertex π_i is created (line 6), having for coordinates the geometric centre of the path (lines 4,5). Thereafter, connections are made between paths that share vertices. For each path $\pi_1 \in \Pi$, the algorithms checks whether any other path $\pi_2 \in \Pi$ has a vertex in common with the former. Given such matching occurs, four different types of situations can emerge:

— π_1 is an undirected path and π_2 is an undirected path;
— π_1 is an undirected path and π_2 is a directed path;
— π_1 is a directed path and π_2 is an undirected path;
— π_1 is a directed path and π_2 is a directed path.

Not all these intersection situations result in the creation of an arc from π_1 to π_2, however. In the undirected-undirected case, the arc $\langle \pi_1, \pi_2 \rangle$ is unconditionally added to $A(I)$ (line 17), which means ultimately that two undirected paths having a vertex in common will have their corresponding vertices in $I(G)$ be joined together by antiparallel arcs.

Path pairings involving directed paths are however more constrained, as the conditions that are applied to them are intended to express the asymmetric nature of one-way streets, which have the particular structural characteristic of not granting access to the street where they begin nor access from the street where they end.

Thus, if a path π_1 is directed (line 12) and has a vertex in common with a directed or undirected path π_2, an arc $\langle \pi_1, \pi_2 \rangle$ is added to $A(I)$ only if the vertex common to both paths is not the tail of the first arc in π_1 (lines 13 and 16). Such condition correspond to the real-world situation in which a one-way street begins at a given one-way or two-way street, in which case the former do not grant access to the latter, since it would imply going in the unauthorized direction. In other words, a car that lies at the beginning of a one-way street (and facing forward, in the authorized direction) simply can't go back to the one-way or two-way street behind without breaking road regulations.

On the other hand, if an undirected or directed path π_1 has a vertex in common with a directed path π_2, an arc $\langle \pi_1, \pi_2 \rangle$ is added to $A(I)$, unless the common vertex is the head of the last arc of π_2. This apparently complex condition simply expresses the fact that a one-way or two-way street does not grant access to the end of a one-way street; in other words, a car on a one-way or a two-way street does not have access to a one-way street that ends on the street it's presently on, since it would imply going in the unauthorized direction.

Of course, such directional considerations and conditions do not emerge in the case of path partitions of non-symmetric road networks (undirected planar straight line graphs), as they do not contain directed paths. In such cases, each arc of the corresponding intersection graph will have its antiparallel counterpart, thus making the directed intersection graph balanced and structurally similar to its underlying graph. On the other hand, all path partitions of non-symmetric road networks will produce unbalanced directed intersection graphs, and thus non-symmetric street networks.

3.4.2 ISOSEQUENTIAL RANDOM GRAPHS

Pertaining to combinatorics and probability theory alike, random graphs first emerged as objects of mathematical and scientific inquiry during the 1950s. In an important 1951 paper published in collaboration with Ray Solomonoff, Rapoport presents the first systematic study of random graphs as well as the demonstration of one of its most well-known properties: a giant connected component containing a finite fraction of all vertices in a random graph emerges exactly when the average degree of that graph exceeds 1.

Following this paper, several important papers on random graph theory were published in the late 1950s and early 1960s (Ford and Uhlenbeck, 1957; Erdős and Rényi, 1959; Austin et al., 1959; Erdős and Rényi, 1961b; Gilbert, 1959). Of all these contributions, however, only those of Erdős and Rényi "introduced the methods which underlie the probabilistic treatment of random graphs" (Bollobás, 2001, p.xii). In a series of articles published between 1959 and 1968 (Erdős and Rényi, 1959, 1961a,b), Erdős and

Rényi used random graphs as probabilistic arguments aimed at proving deterministic graph properties and providing rigorous definitions thereof. Initially used in similar probabilistic proof methods, both cherished and pioneered by Erdős himself (Erdős, 1947, 1959, 1961), random graphs quickly became an object of interest by themselves, as practical applications for them came to be found in all areas involved in complex network modeling and analysis (Newman, 2003a) [9].

Two equivalent variants of the Erdős-Rényi random graph model are usually presented, one defining a random graph $G(n,m)$ as a random element of the probability space consisting of graphs with n vertices and m edges, the other defining a random graph as a probabilistic space over the set of graphs on the vertex set $\{1,...,n\}$ determined by edge probability p:

$$G = (n, Pr[\langle i,j \rangle \in A(G)] = p). \tag{7}$$

In a way, the first model variant consists in blindly choosing a graph from the set of all graphs of same order (number of vertices) and size (number of edges). Given a probability space consisting of all graphs of same order and size, a uniform probability assigned to each graph and a random element G of this probability space, the probability that any graph $H = n, m = G$ equals $\binom{\binom{n}{2}}{n}^{-1}$ (Erdős and Rényi, 1959) [10].

As for the second variant, it consists in assigning, for every pair of vertices in a graph

9. Random networks are indeed useful for the study of complex networks, but it must be stated here that they are in no way representative of real-world networks. More precisely, a random network differs from a real-world network in two ways: "it lacks network clustering or transitivity, and it has an unrealistic Poissonian degree distribution" (Newman, 2003a, p.35)

10. A binomial coefficient $\binom{n}{k}$ returns the number of possible k-combinations in a set of n elements, which is equal to $\frac{n!}{k!(n-k)!}$. As regards to 2-combinations of n elements, as is the case with pairs of vertices in G, the binomial coefficient is $\binom{n}{2}$ and is equal to $\frac{n(n-1)}{2}$. Thus, $\binom{\binom{n}{2}}{n} = \binom{\frac{n(n-1)}{2}}{m}$.

G, a uniform probability of membership in $E(G)$. Thus, while the previous variant can be seen as some kind of random selection, this model easily lends itself to a dynamical interpretation: starting from a set of n isolated vertices, a random graph develops progressively by successively adding edges with with uniform probability p:

> There is a compelling dynamic model for random graphs. For all pairs (i, j) let $x_{i,j}$ be selected uniformly from $[0,1]$, the choices mutually independent. Imagine p going from 0 to 1. Originally, all potential edges are "off". The edge from i to j (which we may imagine as a neon light) is turned on when p reaches $x_{i,j}$, and then stays on. At $p = 1$ all edges are "on" (Alon and Spencer, 2008, p.161-162).

Despite their apparent differences, these two variants are however very much similar. First of all, a dynamic interpretation for the first variant is also possible: beginning with isolated nodes, edges are added with probability $p = \frac{n(n-1)}{2m}$ until m edges are added to $E(G)$, thus making $G(n,m)$ full. Also and most importantly, a graph $G(n,e)$ generated randomly will have very similar properties to a another randomly generated graph $G(n,p)$, with probability distribution $p = e^{-1}\binom{n}{2} = \frac{n(n-1)}{2e}$ (Alon and Spencer, 2008).

Despite their widespread use and proven merits, both variants are however unfit for the present research objectives. First of all, given that the street networks are directed, isosequential random graphs must also be directed. But also, the order and size constraints characteristic of random graph generation procedures are insufficient for the present research purposes, as the in-degree and out-degree sequences of random graphs must be identical to those of the street networks to be analyzed.

The "undirectedness" of Erdős-Rényi random graphs doesn't pose much a problem, since many directed versions have been defined and tested in the literature (Karp, 1990; Cooper and Frieze, 2004; Newman, 2003b; Bollobás, 2001). As regards to probability, the main difference between undirected and directed random graphs lies in the size of the probability space and the value of uniform probability function. The size of the probability space for directed random graphs corresponds to $\binom{\binom{n}{2}}{n}$. As regards to directed graphs, the number of possible arcs for n vertices is equivalent to the number of 2-permutations of n elements, $n(n-1)$, which results in a probability space size equal to $\binom{n(n-1)}{n}$, thus much bigger than the probability space for undirected random graphs.

Research on random directed graphs with fixed in-degree and out-degree sequence has already been conducted and published (Cooper and Frieze, 2004; Bargigli and Gallegati, 2011). The algorithm used in the present research is based on a simple arc-swapping procedure:

Input: $I = \langle V, A \rangle$
Output: *meanCharPathLength*

1 **begin**
2 $\quad list = \emptyset$;
3 $\quad$ **while** $|list| < 10)$ **do**
4 $\quad\quad$ **foreach** $\langle v_1, v_2 \rangle \in A(I)$ **do**
5 $\quad\quad\quad$ $A'(I) = \{x \mid x \in A(I) \wedge head(x) \neq v_1 \wedge tail(x) \neq v_2 \wedge (\{\langle v_1, head(x) \rangle \cup \langle tail(x), v_2 \rangle\} \notin A(I))\}$;
6 $\quad\quad\quad$ $\{v_3, v_4\} = x \mid (\mathscr{R}(A') = x) \wedge (\mathbb{P}(\mathscr{R}(A') = x) = |A'|^{-1})$;
7 $\quad\quad\quad$ $A(G) = \{A(G) \cup \{\langle v_1, v_4 \rangle \cup \langle v_2, v_3 \rangle\}$;
8 $\quad\quad\quad$ $A(G) = \{A(G) \setminus \{\langle v_1, v_2 \rangle \cup \langle v_3, v_4 \rangle\}$;
9 $\quad\quad$ $list = \{list \cup \langle \ell \rangle_{R(I)}\}$;
10 $\quad$ **return** $\sum_{i \in list}^{10} (list(i))10^{-1}$

Algorithm 4: Isosequential Random Graph Generation Algorithm

For each arc $\langle v_1, v_2 \rangle \in A(G)$ (line 4), let A' be the set including all arcs that do not have v_2 as head or v_1 as tail, whose head is not already connected to v_1 and whose tail not already connected to v_2 [11], with selection probability $|A'|^{-1}$ (line 5). The arc $\langle v_3, v_4 \rangle$ then corresponds to a random element of A', in other words the output returned by random function $\mathscr{R}(A')$ with uniform probability A'^{-1} (line 6). Following this random selec-

11. as the following description will make it clear, this condition prevents the creation of loops or parallel arcs in G

tion, both $\langle v_1, v_2\rangle$ and $\langle v_3, v_4\rangle$ are removed from $A(G)$ (line 8) and replaced by $\langle v_1, v_4\rangle$ and $\langle v_3, v_2\rangle$ (line 7). Repeated for all arcs in G, this procedure allows for the randomization of G while preserving both its in-degree and out-degree sequences. Once this randomization procedure is done, the characteristic path length of $R(I)$, $\langle \ell \rangle_{R(I)}\}$, is registered (line 10). This link-swapping-and-characteristic-path-length-calculation procedure is executed 10 times (line 3), after which the average value of the characteristic path lengths of all random isosequential graphs is returned (line 11).

3.4.3 ISOSEQUENTIAL HIGHLY-CLUSTERED ISOSEQUENTIAL GRAPHS

While random graphs can be considered as exemplar disordered structures, lattices such as the regular ring network used in the Watts-Strogatz model represent exemplary cases of reticular orderliness. Such orderliness is tightly linked to the Euclidean nature of the relation connecting vertices together, according to which all vertices connected to a given vertex are also connected to each other. Thus named in reference to Euclid's first Common Notion in Book I of the *Elements*, "Things which are equal to the same thing are also equal to one another" (Heath, 1908, p.155), euclidean relations can be formally defined as relations satisfying $\forall a,b,c((aRb) \wedge (aRc) \rightarrow (bRc)$ [12].

Of course, only complete graphs include arc and edge sets that satisfy the Euclidean condition integrally: most graphs are more or less structured, some vertex neighbours being connected to each other and others not. More importantly, given a set of isosequential graphs, some may show a more lattice-like structure than others, as a greater number of adjacencies satisfy the above-mentioned Euclidean condition. Thus, while the randomization algorithm presented in the previous section generates, for a given pair of in-degree and out-degree sequences, a randomized, isosequential, version of a given graph, it should also be possible, given the same in-degree and out-degree sequences, to generate an isosequential graph whose overall adjacency structure is as

12. It is interesting to note here that euclidean relations are the ones used in the definition and calculation of the average clustering coefficient as defined in Watts and Strogatz (1998)

ordered or as "Euclidean" as possible. Such "latticization" is precisely the objective of the following algorithm.

In order to generate isosequential lattice-like graphs, reference to a new metric or distance function on graph vertices is necessary. In order to distinguish that metric from the earlier quasimetric, this new metric will be denoted by $\rho(x,y)$. As metric, this new real-valued function $\rho(V(G) \times V(G)) \subset \mathbb{R}^+$ satisfies, for any $v_1, v_2, v_3 \in V(G)$, the following conditions:

1. $\rho(v_1,v_2) = 0 \leftrightarrow x = y$
2. $\rho(v_1,v_2) + \rho(v_2,v_3) \leq \rho(v_1,v_3)$
3. $\rho(v_1,v_2) = \rho(v_2,v_1)$

The first axiom, that of identity, states that if the distance between two members of $V(G)$ is null, then these two members are in fact one and the same member. According to the second statement, corresponding to the axiom of triangle inequality, the cumulative distance between a given member of $V(G)$ and two co-members of $V(G)$ is greater than or equal to the distance between these two co-members. Thirdly, the axiom of symmetry states that the distance between an element of $V(G)$ to a co-member of $V(G)$ is the same as the distance from the latter to the former. In addition, given that all vertices of all graphs generated in this research are embedded in the plane, $\forall v \in V(G)(v \in \mathbb{R}^2)$, the distance between two vertices v_1 and v_2 can be defined as follows:

$$(8) \qquad \rho(v_1,v_2) = \sqrt{(first(v_1) - first(v_2))^2 + (last(v_1) - last(v_2))^2}.$$

This definition, which applies the well-known Euclidian distance to vertex coordinates, will be used in order to measure and compare distances between vertices and arc lengths, operations which are essential to the generation of isosequential lattices.

The procedure for isosequential highly-clustered graph generation uses an arc swapping procedure similar to that of the isosequential random graph generation algorithm, but embeds it within a conditional statement and a *while* loop. As in the previous algorithm, the second arc $\langle v_3, v_4 \rangle$ is chosen from the set of arcs whose tail is not v_1, whose head is not v_2, whose head is not already connected to v_1 and whose tail is not already connected to v_2. However, such a choice is not random, as $\langle v_3, v_4 \rangle$ must also be the arc whose head and tail are respectively and jointly closest to v_1 and v_2 (line 8). Such condition allows for newly swapped links to be as short as possible. As for the conditional proposition, it states that arcs are swapped if and only if the sum of the distances between vertices of each potential new pair is less than the sum of the distances between the vertices of the existing pairs in A(G) (line 9). As for the encompassing *while* loop, it allows for the procedure to be restarted for all arcs in G until no swaps are carried out for one whole iteration (line 5). Of course, many iterations are needed before this condition is met; due to this condition, isosequential lattice generation is a rather long process, which may in fact explain "why comparisons with network lattices have not been used in the literature up to this point" (Telesford et al., 2011, p.373). After each iteration of this link-swapping procedure, the average clustering coefficient is calculated for H; if the average clustering coefficient obtained is higher than the maximal average clustering coefficient, then it is registered as the maximal average clustering coefficient (lines 13,14). At the end of the procedure, the highest average clustering coefficient registered is returned (line 15).

While this arc-swapping algorithm may take a long time to execute, the graph it generates is highly clustered: the cumulative shortening of distances between members of each arc in A(H) results in a tightly-knit structure, in which adjacency is strongly correlated with proximity, thus resulting in the emergence of strong local clusters. However, given that such procedure is not deterministic, the probability that two different experiments result in the generation of the same isosequential graph is extremely thin. Also, it must be reminded that the above procedure isn't an optimization algorithm, but a heuristic: it doesn't return the most clustered graph for given in-and-out-degree sequences as output, but rather produces a highly-clustered graph in reasonable time.

Input: $H\langle V,A\rangle \in \mathfrak{R}(G)$
Output: $L(G)$
begin
  MaxACC=0 $swaps = 0$;
  while $swaps > 0$ **do**
    swaps=0;
    foreach $\langle v_1, v_2\rangle \in A(I)$ **do**
      $\langle v_3, v_4\rangle = a \mid (a \in A(I)) \wedge head(a) \neq v_1 \wedge tail(a) \neq v_2 \wedge (\{\langle v_1, head(a)\rangle \cup \langle tail(a), v_2\rangle\} \notin A(I)) \wedge \neg\exists b((b \in A(I)) \wedge (\rho(v_1, head(b)) + \rho(tail(b), v_2)) < (\rho(v_1, v_4) + \rho(v_2, v_3)))$;
      if $\{\langle v_1, v_4\rangle \cup \langle v_2, v_3\rangle\} \notin A(H) \wedge ((\rho(v_1, v_4) + \rho(v_2, v_3)) < (\rho(v_1, v_2) + \rho(v_3, v_4)))$ **then**
        $A(H) = A(H) \cup \{\langle v_1, v_4\rangle, \langle v_2, v_3\rangle$;
        $A(H) = A(H) \setminus \{\langle v_1, v_2\rangle, \langle v_3, v_4\rangle$;
        $swaps = swaps + 1$;
    if $\langle C\rangle_H > MaxACC$ **then**
      $MaxACC = \langle C\rangle_H$
  return MaxACC

Algorithm 5: Isosequential Highly-Clustered Graph Generation Algorithm

CHAPTER IV

RESULTS AND INTERPRETATION

In this chapter, the different results obtained for all 120 street networks are presented and analyzed. First of all, the structural characteristics of the different street networks are presented and compared. Such analysis will allow for a better understanding of the impact of the different parametrization settings such as geometry type (undirected or mixed) and street type (segmental, odonymic, angular) on the resulting networks. Following this, the ω scores of all street graphs are then presented and compared to each other. The aim of such comparative analysis is to assess the impact of road direction and street type on network small-worldness, as measured by the ω metric.

4.1 STRUCTURAL CHARACTERISTICS OF STREET NETWORKS

Comparative structural analysis of all 120 street networks generated in the present research helps to identify some major trends relative to the impact of road direction on street-based road network modeling: while differences in geometry and street type choice have a substantial impact the resulting street networks, parametrization settings for angular street networks (fitness type, threshold) have at most a very minor effect on network structure, thereby calling into question the importance or impact of these parameters on overall street network generation.

4.1.1 SEGMENTAL STREET NETWORKS

Table 6 shows the principal structural characteristics of the segmental street networks of Barnsbury, Clerkenwell and Kensington. In this table, $|V(G)|$ refers to order (number of vertices, i.e. streets), $\langle d\rangle_G$ to average degree, AD to asymmetry degree, r_G to radius (minimal graph eccentricity), d_G to diameter (maximal graph eccentricity), $\langle C\rangle_G$ to average clustering coefficient, and $\langle \ell\rangle_G$ to characteristic path length.

Table 6 – Structural characteristics of segmental street networks

	Barnsbury		Clerkenwell		Kensington	
	Symm.	N-Symm.	Symm.	N-Symm.	Symm.	N-Symm.
$\lvert V(G)\rvert$	5537	5614	10198	10342	8886	8954
$\langle d\rangle_G$	8.11	6.14	8.32	5.91	8.43	6.05
AD	-	0.24	-	0.30	-	0.29
r_G	50	54	55	63	53	56
d_G	91	101	102	114	97	112
$\langle C\rangle_G$	0.46	0.36	0.46	0.33	0.45	0.32
$\langle \ell\rangle_G$	36.41	40.21	44.07	49.64	39.86	44.46

Analysis of the structural characteristics of segmental street networks proves useful on various grounds. First of all, the fact that these models don't allow streets to extend beyond intersections should allow them to act as a kind of null hypothesis, allowing for a better assessment of the impact of road asymmetry on both odonymic and angular street networks. But also, their structure gives an accurate picture of the asymmetry degree of the three neighbourhoods considered in this study: by having one node for each direction-homogeneous segment (no changes in direction type between intersections), segmental street graphs can highlight connection patterns between road segments and most importantly formally express the impact of road direction on street accessibility and turn restrictions [1]. Indeed, when the adjacency between two nodes in a segmental

1. In this sense, segmental street networks can play a structural role similar to that of line graphs as defined in subsection 3.4.1 (Winter, 2002; Caldwell, 1961)

street network is symmetric, that is, when there are two antiparallel arcs between these nodes, then the two segments corresponding to these nodes have access to one another; however, when there is only one link between two nodes, it means the accessibility relation between the two corresponding segmental streets is asymmetric and that one of them is necessarily made of one-way road sections.

In light of these considerations, the asymmetry degree of a segmental street network is thus the proportion of arcs that do not have an antiparallel counterpart. In this regard, the segmental street graphs of the three neighbourhoods under study present rather similar structural properties. As shown in Table 6, 23.9% of all accessibility relations between Barnsbury streets are asymmetric, while the asymmetry for the neighbourhoods of Clerkenwell and Kensington are even higher, with 29.53% and 28.81% respectively. Given that average total degree values for each non-symmetric segmental street network, in the same order as before, are respectively 1.97, 2.41, and 2.38 degrees lower than those of corresponding symmetric networks, it is fair to say that every street node of every non-symmetric street network loses around two connections to other streets compared to the symmetric street graph of the same neighbourhood. Given these numbers, three situations can arise:

— a segmental street part of the out-neighbourhood of S isn't part of its in-neighbourhood anymore (it can be reached by S but cannot grant access to S anymore);
— a segmental street part of the in-neighbourhood of S isn't part of its out-neighbourhood anymore (it can grant access to S but cannot be reached by it anymore);
— a segmental street part of the in-neighbourhood and out-neighbourhood of S in symmetric context is excluded from both in non-symmetric context (as in the case with one-ways going in opposite directions).

Given that such situations apply to every street node of almost every segmental street network (the decrease in average total degree for the segmental street network of Barnsbury is just under 2), it must be concluded that road asymmetry awaits at every turn and at the end of every segment.

Another striking fact is the difference in order between the symmetric and non-symmetric segmental street networks for each of the three London neighbourhoods. Barnsbury has 77 more segmental streets in its non-symmetric street network, while Clerkenwell and Kensington respectively have 144 and 68 more segmental street nodes in their non-symmetric street networks. Normally, one would think that partitioning a road network in segments, irrespective of their direction type (one-ways or both-ways) would generate the same number of nodes. However, it must be reminded that for all modeling procedures used in this research, direction supervenes on street membership: two road sections cannot be part of the same street path if they are direction-incompatible, that is, if one is a one-way and the other a both-way or both are one-ways pointing in different directions. It has also been previously said that many roads in the London area change direction type between intersections in order to prevent access from one direction and thus restrain local traffic. Differences in street network order is a direct result of this, as direction-changing road sections between intersections are converted into distinct segmental street network nodes. Also, since supervenience of road direction on street membership equally applies to odonymic and angular street networks, direction-incompatible road sections between intersections will also and necessarily be part of distinct odonymic or angular street paths, regardless of their respective odonyms or the deflection angle they form.

As regards to their radius, diameter, clustering coefficient and characteristic path length, the segmental street networks of the three neighbourhoods present similar properties. While radii and diameters for all segmental street networks are much longer than those of odonymic and angular street networks, symmetric or non-symmetric alike, the radii and diameters of non-symmetric segmental networks are always longer than those of their symmetric counterparts. This difference can be explained by the presence of one-way segmental streets, which prevent street connections that would otherwise shorten path lengths; the same explanation goes for the discrepancy between the characteristic path lengths of symmetric and non-symmetric street networks. As for the clustering coefficient, it is rather high for all street networks, irrespective of their direction type.

This is to be expected of segmental street networks, as it reflects the highly-ordered nature of grid configurations characteristic of urban networks. Here again, a notable difference can be observed between symmetric segmental street networks and their non-symmetric counterparts. Once more, road direction is responsible for this discrepancy, as the presence of asymmetric street connections (those that don't have antiparallel counterparts) translates into lower local clustering coefficients, which in turns affects the average clustering coefficient.

In all its aspects, analysis of symmetric and non-symmetric segmental street networks already shows the strong impact road direction has on the structural characteristics of urban networks, despite the fact that street paths extend only up to intersections. Given this, it can be expected that the effect of road direction on odonymic and segmental path construction procedures as well as on the resulting street networks, will be no less significant.

4.1.2 ODONYMIC STREET NETWORKS

As Table 17 shows, odonymic street networks appear more symmetric than segmental street networks. First, only a fifth of all odonymic street connections are unidirectional (19.52% for Barnsbury, 23.74% for Clerkenwell and 23.21% for Kensington), which is lower than the asymmetry degrees of segmental street networks. Moreover, the average degree values of non-symmetric street networks is still lower than in symmetric context, but the decrease is less important as the one observed in the case of segmental streets.

This damping of the "road direction effect" on asymmetry degree and average total degree doesn't necessarily entail a lower impact of road direction on street structure, however. On the contrary, many aspects tend to indicate that the impact of road direction is even stronger on odonymic street networks than on their segmental counterparts. For once, discrepancies in network order are much more pronounced, as the non-symmetric odonymic street networks of Barnsbury, Clerkenwell and Kensington respectively count 346, 744 and 650 more nodes than their symmetric counterpart. This difference represents an increase of 16.08%, 19.55% and 17.63% compared to symmetric network order respectively, which is significantly more than in the case of segmental street graphs (with 1.39% for Barnsbury, 1.41% for Clerkenwell and 0.77% for Kensington). This increase is once again the result of supervenience of road direction on street membership, as the direction compatibility requirement not only applies to road sections between intersections only, but also to road sections at road crossings: two synonymic road sections meeting at an intersection cannot be part of the same street path if one is a one-way and the other is not or if both sections are one-ways pointing in opposite directions. The structural impact of road direction on street networks thus seems greater when street structure extends beyond intersections, as is the case with odonymic street paths.

Table 7 – Structural characteristics of odonymic street networks

	Barnsbury		Clerkenwell		Kensington	
	Symm.	N-Symm.	Symm.	N-Symm.	Symm.	N-Symm.
$\|G\|$	2152	2498	3806	4540	3037	3687
$\langle d \rangle$	6.93	5.76	7.21	5.74	7.47	5.97
AD	-	0.20	-	0.24	-	0.23
r_G	9	13	12	17	13	20
d_G	17	25	24	38	23	36
$\langle C \rangle_G$	0.16	0.14	0.17	0.14	0.18	0.15
$\langle \ell \rangle_G$	7.43	10.48	9.55	13.54	9.36	13.39

Another case attesting to the stronger impact of road direction on odonymic street networks involves path-related characteristics. First, an augmentation of 44%, 41.67% and 53.85% can be observed for the radii of the non-symmetric odonymic street networks of Barnsbury, Clerkenwell and Kensington compared to their symmetric counterparts, which is a lot higher than the 8%,14.55% and 24.53% increases observed in segmental context. Similar discrepancies can be observed in the case of diameters, with increases of 47.06%, 58.33% and 56.52% for Barnsbury, Clerkenwell and Kensington non-symmetric street networks, compared to augmentations of 10,99%, 11,76% and 15,46% for non-symmetric segmental ones. Given these important discrepancies and the fact that the calculation of network radii, diameters and characteristic path lengths are all based on the computation of shortest path lengths, the fact that equivalent augmentations can be observed in the characteristic path lengths of non-symmetric odonymic networks should come as no surprise. Indeed, the characteristic path lengths of non-symmetric odonymic street networks are more than 40% higher than those of odonymic symmetric networks, with increases of 41.05% for Barnsbury, 41.78% for Clerkenwell and 43.06% for Kensington. Given that the same increase for the non-symmetric, segmental street networks of Barnsbury, Clerkenwell and Kensington is respectively of 10.44%, 12.64% and 11.54%, the impact of road direction on the characteristic path length of odonymic street networks is almost four times as great as the same impact on segmental street networks. Needless to say, it is to be expected that a difference as strong at this one translates into equally important discrepancies in network small-worldness, as indicated by ω scores. While it is true that the average clustering coefficients of segmental street networks (with increases of 21.74%, 28.26% and 28.89% for Barnsbury, Clerkenwell and Kensington respectively) is more sensitive to road direction than those of odonymic street networks (with increases of 12.25%, 17.65% and 16.67%, in the same order as before), this sole exception to the general structural pattern cannot alone be sufficient to counterbalance the profound effect discrepancies in path length might have on network small-worldness.

4.1.3 ANGULAR STREET NETWORKS

Analysis of the structural characteristics of angular street networks is trickier than that of segmental and odonymic street networks, if only because of the amount of data to analyze. Indeed, differences between symmetric and non-symmetric networks for this street graph type need to be considered and evaluated on different levels, depending not only on the neighbourhood modelled, but also on the angular fitness type and threshold value of the networks under consideration.

As regards to asymmetry degree, Tables 8 to 16 show that the number of unidirectional adjacency relations between streets is slightly lower in angular street networks than in

Table 8 – Structural characteristics of Barnsbury simple angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$
20	1842 / 6.96	7 / 14	0.20 / 5.43	2056 / 5.76	0.14	11 / 20	0.14 / 8.18
30	1827 / 6.94	7 / 13	0.20 / 4.83	2046 / 5.76	0.14	11 / 20	0.14 / 7.94
40	1820 / 6.93	7 / 13	0.20 / 4.76	2038 / 5.74	0.14	11 / 19	0.14 / 7.92
50	1811 / 6.95	7 / 13	0.20 / 4.73	2036 / 5.75	0.14	10 / 18	0.14 / 7.76
60	1807 / 6.92	7 / 14	0.21 / 4.75	2034 / 5.73	0.14	11 / 21	0.14 / 8.10
70	1807 / 6.82	7 / 13	0.21 / 4.85	2033 / 5.70	0.14	11 / 21	0.14 / 8.41

Table 9 – Structural characteristics of Barnsbury relative angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$
20	1848 / 6.99	8 / 14	0.20 / 5.55	2056 / 5.76	0.14	11 / 20	0.14 / 8.18
30	1833 / 6.98	8 / 15	0.20 / 5.40	2046 / 5.75	0.14	11 / 20	0.14 / 7.93
40	1828 / 6.98	7 / 14	0.19 / 5.27	2039 / 5.74	0.14	11 / 20	0.14 / 7.86
50	1822 / 6.98	7 / 13	0.19 / 5.20	2037 / 5.74	0.14	10 / 19	0.14 / 7.78
60	1820 / 6.98	7 / 13	0.19 / 5.19	2036 / 5.74	0.14	10 / 19	0.14 / 7.77
70	1819 / 6.98	7 / 13	0.19 / 5.19	2035 / 5.75	0.14	10 / 19	0.14 / 7.77

Table 10 – Structural characteristics of Barnsbury mutual angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$
20	1848 / 6.99	8 / 14	0.20 / 5.55	5.78 / 5.76	0.14	11 / 20	0.14 / 8.18
30	1833 / 6.98	8 / 15	0.20 / 5.42	5.75 / 5.75	0.14	11 / 20	0.14 / 7.93
40	1829 / 6.98	7 / 14	0.19 / 5.29	5.74 / 5.74	0.14	11 / 20	0.14 / 7.85
50	1825 / 6.99	7 / 13	0.19 / 5.21	5.74 / 5.74	0.14	10 / 18	0.14 / 7.77
60	1824 / 6.99	7 / 13	0.19 / 5.21	5.74 / 5.74	0.14	10 / 18	0.14 / 7.76
70	1824 / 6.99	7 / 13	0.19 / 5.21	5.75 / 5.75	0.14	10 / 18	0.14 / 7.76

the case of odonymic street networks. Particularly noteworthy is the low variability of degree values, regardless of fitness type or threshold value. Rounded to the nearest hundredth, the asymmetry degree of Barnsbury street graphs stays at 0.14, while it oscillates between 0,17 and 0,18 for the different Clerkenwell networks and between 0.16 and 0.17 in the case of Kensington street networks.

Here again, the supervenience of road direction on street membership for non-symmetric street networks results in an augmentation of the number of streets. For simple, relative and mutual street networks respectively, such increase amounts on average to:

— 12.18% (simple), 11.66% (relative) and 11.56% (mutual) in the case of Barnsbury,
— 14.18% (simple), 13.79% (relative) and 13.66% (mutual) for Clerkenwell, and
— 13.58% (simple), 13.04% (relative) and 12.94% (mutual) for Kensington.

Thus, the impact of the supervenience of street membership on the number of angular streets is less than its impact on the number of odonymic streets, as it translates into an augmentation 4% to 6% inferior to that of non-symmetric odonymic street networks. This difference may be due to the fact that angular one-way streets may reach longer than odonymic ones: in the three neighbourhoods of London under study, series of continuous one-way roads with small deflection angles often change names after a few blocks (for example, Tollington Road becoming Camden Road after Holloway Road and Parkhurst Road becoming Seven Sisters Road after Holloway Road), a fact which results in the creation of distinct odonymic streets that would otherwise be united in the same angular street path. Of course, this line of reasoning also applies to series of continuous, low-deflection two-way roads that change name after intersections, but the fact that angular street networks have lower asymmetry degrees than odonymic street networks, hence less one-ways streets, gives weight to the former scenario.

Unlike both asymmetry degree and order, the "road direction effect" has a slightly stronger impact on the average total degree values of angular street networks compared to those of odonymic street networks. Indeed, the average total degree values of

Table 11 – Structural characteristics of Clerkenwell simple angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle \ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle \ell\rangle_G$
20	3241 / 7.37	8 / 16	0.21 / 6.08	3680 / 5.77	0.18	11 / 21	0.15 / 8.97
30	3201 / 7.35	8 / 16	0.22 / 5.64	3648 / 5.78	0.18	11 / 21	0.15 / 8.63
40	3189 / 7.32	7 / 14	0.22 / 5.59	3637 / 5.78	0.17	11 / 21	0.15 / 8.68
50	3178 / 7.30	7 / 14	0.22 / 5.49	3631 / 5.78	0.17	11 / 21	0.15 / 8.65
60	3168 / 7.30	8 / 15	0.22 / 5.48	3629 / 5.77	0.17	11 / 21	0.16 / 8.64
70	3162 / 7.28	8 / 15	0.22 / 5.48	3627 / 5.75	0.17	12 / 22	0.15 / 9.18

Table 12 – Structural characteristics of Clerkenwell relative angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle \ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle \ell\rangle_G$
20	3248 / 7.39	9 / 16	0.21 / 6.50	5.78 / 5.78	0.18	11 / 21	0.15 / 8.97
30	3210 / 7.38	8 / 15	0.21 / 6.17	5.78 / 5.78	0.18	11 / 21	0.15 / 8.82
40	3204 / 7.39	8 / 15	0.21 / 6.12	5.79 / 5.79	0.17	11 / 21	0.16 / 8.67
50	3191 / 7.39	8 / 15	0.21 / 6.06	5.79 / 5.79	0.17	11 / 21	0.16 / 8.61
60	3186 / 7.39	8 / 15	0.21 / 6.01	5.79 / 5.79	0.17	11 / 21	0.16 / 8.57
70	3184 / 7.39	8 / 15	0.21 / 6.01	5.80 / 5.80	0.17	11 / 21	0.16 / 8.57

Table 13 – Structural characteristics of Clerkenwell mutual angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle \ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle \ell\rangle_G$
20	3248 / 7.39	9 / 16	0.21 / 6.50	3683 / 5.78	0.18	11 / 21	0.15 / 8.97
30	3210 / 7.38	8 / 15	0.21 / 6.17	3651 / 5.78	0.18	11 / 21	0.15 / 8.82
40	3204 / 7.39	8 / 15	0.21 / 6.12	3640 / 5.79	0.17	11 / 21	0.16 / 8.67
50	3195 / 7.39	8 / 15	0.21 / 6.07	3634 / 5.79	0.17	11 / 21	0.16 / 8.61
60	3194 / 7.39	8 / 15	0.21 / 6.02	3633 / 5.79	0.17	11 / 21	0.16 / 8.57
70	3194 / 7.39	8 / 15	0.21 / 6.02	3632 / 5.80	0.17	11 / 21	0.16 / 8.57

the non-symmetric angular street networks of Barnsbury, Clerkenwell and Kensington are respectively 1.22, 1.59 and 1.61 degrees higher than those of symmetric networks, which is more than the decreases of 1.17, 1.47 and 1.50 observed for odonymic networks. Given the comparatively smaller change in both asymmetry degree and order for angular networks compared to odonymic networks, this relative rise in average degree means that the "road direction effect" on angular street networks results in smaller street loss but in a comparatively bigger decrease in street connections, the latter however resulting in a higher proportion of bidirectional street connections than in non-symmetric, odonymic context.

As regards to path-related structural properties, angular street networks stand out in a few areas. For each neighbourhood, the road direction effect on radius length results increases of:

- 54.76% (simple), 43.18% (relative) and 43.18% (mutual) for Barnsbury,
- 45.65% (simple), 34.69% (relative) and 34.69% (mutual) for Clerkenwell, and
- 47.92% (simple), 30.77% (relative) and 30.77% (mutual) for Kensington.

Except for simple street networks and Barnsbury angular street networks, the augmentations are significantly lower than those observed for non-symmetric odonymic street networks. This smaller effect of road asymmetry in angular context is even more obvious in the case of the average diameter, with rises of:

- 48.75% (simple), 42.68% (relative) and 39.02% (mutual) for Barnsbury,
- 41.11% (simple), 38.46% (relative) and 38.46% (mutual) for Clerkenwell, and
- 35.48% (simple), 20.19% (relative) and 20.19% (mutual) for Kensington.

Indeed, with the sole exception of the simple and relative street networks of Barnsbury, the rise in diameter for non-symmetric angular networks compared to their symmetric counterpart is often 10% inferior than those of non-symmetric odonymic networks; the increase for the relative and mutual street networks of Kensington is even less than half that observed for non-symmetric odonymic networks.

As regards to the impact of road direction on the increase in averaged characteristic path length for the non-symmetric angular street networks, the results for each neighbourhood are:

— 64.6% (simple), 48.71% (relative) and 49.19% (mutual) for Barnsbury,
— 56.25% (simple), 41.61% (relative) and 41.49% (mutual) for Clerkenwell, and
— 43.4% (simple), 27.89% (relative) and 27.92% (mutual) for Kensington.

At first glance, the increase in average characteristic path length for the non-symmetric simple networks of Barnsbury and Clerkenwell compared to that of their symmetric counterparts (41,05% and 41,78% respectively) stand out from the rest. In both cases, such a high increase is probably the result of the low characteristic path lengths of the 6 symmetric simple networks: in the case of Barnsbury, five of these are the only angular street networks presenting characteristic path lengths under 5 average steps; as for Clerkenwell, the simple networks with threshold superior to 20 degrees are the only angular street networks presenting a characteristic path length lower than 6 average steps. At the other extreme, increases in characteristic path length for the relative and mutual networks of Kensington is more than 15% lower than those observed for the non-symmetric odonymic network of the same neighbourhood; this shouldn't come as a surprise, however, as the average diameter of those networks follows the same trend. Apart from these cases, increases in characteristic path length of non-symmetric angular street networks are fairly similar to those observed for non-symmetric odonymic street networks.

Finally, in the case of increase in average clustering coefficient for the non-symmetric angular street networks, the averaged values for each street type are:

— 31.15% (simple), 27.59% (relative) and 27.59% (mutual) for Barnsbury,
— 30.53% (simple), 25.4% (relative) and 25.4% (mutual) for Clerkenwell, and

Table 14 – Structural characteristics of Kensington simple angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$
20	2717 / 7.66	8 / 15	0.21 / 6.12	3082 / 6.04	0.17	12 / 23	0.16 / 8.20
30	2691 / 7.66	8 / 16	0.21 / 5.95	3051 / 6.05	0.17	12 / 22	0.16 / 8.02
40	2680 / 7.66	8 / 16	0.21 / 5.59	3038 / 6.06	0.17	12 / 20	0.16 / 7.96
50	2667 / 7.63	8 / 15	0.21 / 5.54	3031 / 6.06	0.17	12 / 20	0.16 / 7.91
60	2659 / 7.60	8 / 16	0.22 / 5.48	3025 / 6.05	0.17	11 / 20	0.17 / 7.99
70	2650 / 7.56	8 / 15	0.23 / 5.10	3018 / 6.03	0.16	12 / 21	0.17 / 8.36

Table 15 – Structural characteristics of Kensington relative angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle\rvert$	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$
20	2722 / 7.67	8 / 16	0.21 / 6.33	3083 / 6.04	0.17	12 / 23	0.16 / 8.21
30	2699 / 7.69	8 / 16	0.21 / 6.17	3053 / 6.06	0.17	12 / 22	0.16 / 8.01
40	2691 / 7.70	9 / 18	0.21 / 6.20	3041 / 6.06	0.17	11 / 20	0.17 / 7.87
50	2682 / 7.70	9 / 18	0.21 / 6.16	3032 / 6.07	0.17	11 / 20	0.16 / 7.82
60	2681 / 7.69	9 / 18	0.21 / 6.16	3028 / 6.05	0.16	11 / 20	0.16 / 7.82
70	2679 / 7.69	9 / 18	0.21 / 6.16	3024 / 6.07	0.16	11 / 20	0.16 / 7.82

Table 16 – Structural characteristics of Kensington mutual angular street networks

	Symmetric			Non-Symmetric			
	$\lvert G\rvert$ / $\langle d\rangle$	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$	$\lvert G\rvert$ / $\lvert A(G)\rvert$	AD	r_G / d_G	$\langle C\rangle_G$ / $\langle\ell\rangle_G$
20	2722 / 7.67	8 / 16	0.21 / 6.33	3083 / 6.04	0.17	12 / 23	0.16 / 8,21
30	2699 / 7.69	8 / 16	0.21 / 6.17	3053 / 6.06	0.17	12 / 22	0.16 / 8.01
40	2693 / 7.70	9 / 18	0.21 / 6.20	3041 / 6.06	0.17	11 / 20	0.17 / 7.87
50	2687 / 7.70	9 / 18	0.21 / 6.16	3034 / 6.06	0.17	11 / 20	0.16 / 7.83
60	2686 / 7.70	9 / 18	0.21 / 6.16	3030 / 6.07	0.16	11 / 20	0.16 / 7.82
70	2686 / 7.70	9 / 18	0.21 / 6.16	3025 / 6.07	0.16	11 / 20	0.16 / 7.82

— 24.03% (simple), 23.02% (relative) and 23.02% (mutual) for Kensington.

Regardless of angular fitness type, these increases are well above those observed for non-symmetric odonymic networks. In the case of Barnsbury simple networks, the increase is almost three times higher than that for odonymic networks (12,25%); as for relative and mutual networks of the same neighbourhood, the difference is more than twice the one observed in non-symmetric, odonymic networks. The smallest differences are still noteworthy, with increases around 8% and 7% for both relative and mutual street networks of Clerkenwell and Kensington respectively.

A last but important point must be made regarding angular street networks. As shown in Table 17, structural properties of the different angular street types are relatively homogeneous. Mean (μ) and relative standard deviation values ($\sigma_r(\%)$) for the different structural properties of all symmetric and non-symmetric angular networks, irrespective of neighbourhood, angular fitness type and threshold are shown in Table 17

Table 17 – μ and $\sigma_r(\%)$ values for symmetric and non-symmetric network structural properties

		Barnsbury		Clerkenwell		Kensington	
		Symm.	N-Symm.	Symm.	N-Symm.	Symm.	N-Symm.
$\|G\|$	μ	1825.94	2041.39	3200.39	3649.89	2688.39	3042.89
	$\sigma_r(\%)$	0.60	0.38	0.77	0.75	0.72	0.68
$\langle d \rangle$	μ	6.96	5.74	7.37	5.78	7.67	6.06
	$\sigma_r(\%)$	0.57	0.17	0.41	0.17	0.52	0.17
ε	μ	7.22	10.61	8.00	11.06	8.44	11.50
	$\sigma_r(\%)$	5.96	4.71	6.13	2.17	6.04	4.43
E	μ	13.56	19.44	15.11	21.06	16.72	20.89
	$\sigma_r(\%)$	5.16	5.04	3.84	1.13	7.36	5.89
$\langle C \rangle_G$	μ	0.20	0.14	0.21	0.16	0.21	0.16
	$\sigma_r(\%)$	3.50	0.00	2.17	3.29	2.41	2.59
$\langle \ell \rangle_G$	μ	5.17	7.94	5.97	8.73	6.01	7.97
	$\sigma_r(\%)$	5.22	2.39	5.40	2.06	5.82	2.13

What is most striking about these different statistics is how low deviation for angular streets is compared to the average values for each structural property, irrespective of fitness type and threshold. The highest deviation, that of the diameters for symmetric Kensington angular networks, represents only 7,36% of the mean value, which is still more than 1% higher than the second-highest relative standard deviation recorded. In the case of non-symmetric angular networks, the only deviations above 5% are those corresponding to the mean diameters of Barnsbury (5,04%) and Kensington (5,89%) angular street networks. Aside from non-path-related properties (radius, diameter, characteristic path length), the largest relative standard deviation for any structural property, belonging to non-symmetric angular networks of Barnsbury, is slightly above 3% (3,5%). In addition, all standard deviations in graph size and average degree are well below 1% of their respective mean value. Finally and most surprisingly, the average clustering coefficient for the non-symmetric odonymic network of Barnsbury, once rounded to the nearest hundredth, doesn't vary at all. Thus, there is globally much less variation in structural property values between networks of different angular street type and threshold value than between symmetric and non-symmetric angular networks (with the sole exception of the variation in order between non-symmetric and symmetric segmental graphs). But most importantly, the taking into account of road direction results in even stronger homogenization, as relative standard deviation is at its lowest in non-symmetric context.

Given all this, the question of the relevance of the use of different fitness types and threshold values in assessing of the impact of road direction on street-based network modeling can certainly be questioned. However, analysis of the ω scores of the different angular networks is needed before passing judgment on that matter.

4.1.4 STRUCTURAL IMPACT: CONCLUSIONS

As the above analysis shows, road direction impacts ubiquitously, albeit differently, on segmental, odonymic and angular street networks. At first glance, structural fluctuations due to the taking into account of road direction are similar across all street networks: indeed, order (number of streets) and path-related characteristics (radius, diameter and characteristic path length) increase for all street networks when road direction is taken into account, while average total degree, average clustering coefficient diminishes in non-symmetric context.

However, beyond these general patterns lies a wide variation in impact between street networks and neighbourhoods. Given the data collected from all street networks, the most tenable hypothesis would be that these variations result from three factors: choice of neighbourhood, asymmetry degree and street type. While it seems self-evident that the choice of a different neighbourhood may result in changes in structural properties, it is nonetheless important to stress the differences in results between the neighbourhoods of Barnsbury on one side and those of Clerkenwell and Kensington on the other. Indeed, structural properties for Barnsbury differ significantly from those of the two other neighbourhoods, both in symmetric and non-symmetric context: when it comes to order, size, radius, diameter, average clustering coefficient or characteristic path length, results for Barnsbury are always slightly lower than those of the two other neighbourhoods. Given that such distinctiveness is probably due to smaller neighbourhood size and lower asymmetry degree, it seems important to stress here that such discrepancies are likely to be reflected in the ω scores of the different street networks.

As regards to both choice in street type and discrepancies in asymmetry degree, relative standard deviation for the average asymmetric degree values of non-symmetric segmental (28%), odonymic (23%) and angular (16%) street graphs of all three neighbourhoods represents more than a fourth (27%) of the mean asymmetric degree value of all street networks. Given this, both asymmetry degree and choice of street type are expected to be the main discriminating factors involved in ω score variability among the different street networks. Moreover, given the importance of the structural discrepancies observed for the different street networks and linked to these two factors, equally important discrepancies are to be expected.

4.2 ω SCORES FOR STREET NETWORKS

The ω scores of all 120 different street networks generated in this study are shown in Table 18. Taken together, these scores provide an interesting picture of the impact of road asymmetry on street-based modeling of road networks. With respect to street types, segmental, odonymic and angular street networks present widely diverging ω scores, which confirms the importance and impact of street type choice on the resulting street networks as well as their distinctiveness in relation to traditional, metrically-faithful, road network models. As regards to road direction, significant differences between ω scores for symmetric and non-symmetric street networks can be observed. Given that the same tendencies can be observed for the different neighbourhoods, these results help stress the necessity of taking into account road direction for street-based modeling and analysis.

4.2.1 SEGMENTAL STREET NETWORKS

First and most strikingly, the ω scores of segmental street networks clearly stand out from those of all other street networks in two different ways. First, whereas the ω scores of all other street networks are positive, those of both symmetric and asymmetric segmental networks are exclusively negative, thus indicating their strong orderly and lattice-like structure. But also and equally importantly, segmental street graphs have by far the most extreme scores, that is, those that deviate the most from the ω mid-point (0), corresponding to ideal small-worldness. Both characteristics, which taken together are indicative of extremely ordered, "tightly-knit" networks, result from the fact that the average clustering coefficient and characteristic path length of segmental street networks are extremely high compared to those of the corresponding isosequential highly-clustered and random graphs respectively. However, given that such structural characteristics are the direct result of the metric-preserving geographical embedding of segmental networks, which entails strong limitations on both node degree and long-range connections (Barthélémy, 2011), ω scores of segmental street networks shouldn't come as surprising.

Table 18 – ω Scores for Symmetric and Non-Symmetric Street Networks

		Barnsbury		Clerkenwell		Kensington	
		Symm.	N-Symm.	Symm.	N-Symm.	Symm.	N-Symm.
Segmental		-0.76	-0.81	-0.77	-0.82	-0.74	-0.75
Odonymic		0.35	0.23	0.20	0.06	0.20	0.07
Angular Best	20	0.43	0.31	0.36	0.27	0.35	0.27
	30	0.50	0.32	0.40	0.26	0.37	0.29
	40	0.50	0.34	0.41	0.27	0.42	0.30
	50	0.51	0.35	0.41	0.27	0.41	0.29
	60	0.48	0.32	0.40	0.28	0.40	0.28
	70	0.47	0.30	0.41	0.23	0.43	0.25
Angular Mutual	20	0.43	0.32	0.32	0.26	0.33	0.27
	30	0.44	0.33	0.36	0.27	0.35	0.28
	40	0.47	0.35	0.35	0.27	0.35	0.29
	50	0.47	0.34	0.35	0.27	0.35	0.30
	60	0.47	0.34	0.37	0.28	0.34	0.28
	70	0.47	0.34	0.37	0.28	0.35	0.28
Angular Relative	20	0.43	0.31	0.33	0.26	0.33	0.28
	30	0.44	0.32	0.36	0.25	0.35	0.28
	40	0.47	0.32	0.36	0.27	0.34	0.29
	50	0.48	0.35	0.36	0.27	0.34	0.29
	60	0.48	0.34	0.37	0.27	0.35	0.29
	70	0.47	0.34	0.36	0.27	0.35	0.29

As regards to the evaluation of the "road direction effect" on segmental street networks, the impact is subtle yet clear: ω scores of all three non-symmetric segmental networks are slightly closer to the lower bound of -1 than their symmetric counterparts, which is indicative of more orderly and lattice-like networks. Given the differences in average clustering coefficient and characteristic path length values for both types of segmental street networks, discrepancies in their respective ω scores was to be expected. On closer analysis, the better ω scores observed for non-symmetric segmental street networks are due to the greater proximity in average clustering coefficient between the latter and their corresponding isosequential high-clustered graphs, which brings the ω scores of non-symmetric networks closer to the lower bound of -1.

Thus, as a first assessment of the performance, accuracy and appropriateness of the ω metric, analysis of segmental street networks shows that ω scores, either through comparison with street networks of different street type or between symmetric and non-symmetric networks of same street type, allow for an effective comparative analysis of network orderliness.

4.2.2 ODONYMIC STREET NETWORKS

As for odonymic networks, their ω scores are clearly the best of all street graphs; this is even more so in the case of non-symmetric odonymic networks, whose scores are extremely close to the ideal small-worldness score of 0, with 0.06 for Clerkenwell and 0.07 for Kensington. These exceptional small-worldness scores mean that odonymic street networks achieve a better balance between local and global network efficiency, as measured by their average clustering coefficient and characteristic path length.

At first glance, this better small-worldness of odonymic street networks might appear surprising, given that for each neighbourhood, the average clustering coefficient and characteristic path length of odonymic networks are respectively lower and higher than those of the angular networks of the same neighbourhood. Closer analysis however reveals that random graphs isosequential to odonymic street networks also have a significantly higher characteristic path than those isosequential to angular street networks; but more importantly, the average clustering coefficients of the highly-clustered, lattice-like graphs corresponding to odonymic street networks are much closer to those of the latter, which is the sign of a relatively more ordered structure. Thus, odonymic street networks are globally and structurally much closer to the midpoint between the randomness and orderliness of their respective random and lattice-like counterparts than angular and segmental networks. As for "the road direction effect", it only exacerbates this distinction, as it brings non-symmetric odonymic street closer to the structural equilibrium point of ideal small-worldness.

From a cognitive point of view, these results are highly suggestive. Informationally speaking, street labeling is a strategy that pertains both to chunking and cueing: it groups a set of segments together by creating a new lexical concept, with its own unique identity and meaning, which can then be quickly referred to in any wayfinding, route planning or communication task by anyone familiar with the street network. And as the excellent ω scores of asymmetric nominal topologies seem to suggest, these different space-related activities can all be performed on the basis of structural representations that are extremely efficient information-wise. More will be said on that matter in the conclusion

4.2.3 ANGULAR STREET NETWORKS

As regards to the different angular street networks, symmetric ω scores are on the outer edge of the small-world range and lean strongly towards the disorderliness of random-like network structures. This is especially true in the case of the Barnsbury street networks, as ω scores for simple angular street networks stretch to the limit of small-worldness and even beyond. From a small-worldness perspective, this structural imbalance of angular networks is essentially due to their characteristic path lengths, which is much closer to those of their corresponding isosequential randoms graph than their average clustering coefficients are to those of their corresponding isosequential lattice-like graphs.

As regards to the "road direction effect", the small-worldness impact of non-asymmetry is comparable to the one observed for odonymic street networks, as the taking into account of road direction has the effect of bringing the ω scores of angular street networks closer to 0. However, with the sole exception of the simple street network of Clerkenwell with 70^o angular threshold, which has an ω score equal to that of the non-symmetric odonymic street network of Barnsbury, all angular networks have lower ω scores than their odonymic counterparts. In the case of the simple street networks of Barnsbury, road direction even "saves the day", as the taking into account of road direction makes them all part of the small-world zone, which isn't the case in symmetric context.

Finally, regarding the structural homogeneity issue raised in the previous section, relative standard deviation of ω scores for angular networks compares well with the variability observed for other structural characteristics, as the Table 19 shows.

Table 19 – μ and $\sigma_r(\%)$ values of symmetric and non-symmetric angular network ω scores

	Barnsbury		Clerkenwell		Kensington	
	Symm.	N-Symm.	Symm.	N-Symm.	Symm.	N-Symm.
μ	0.47	0.33	0.37	0.27	0.36	0.28
$\sigma_r(\%)$	5.11	4.55	7.30	4.44	8.61	4.29

While the ω scores of non-symmetric networks are significantly closer to 0 than those of their symmetric counterparts, the choice of angular fitness type and threshold value seems to have but a minor impact on small-worldness, as the relative standard deviation values for the averaged ω scores of all simple, relative and mutual networks for each neighbourhood represent less than 9% of the ω average. The taking into account of road direction dampens variability in ω scores for angular street networks even further, as all relative standard deviation values for non-symmetric angular networks is within 5% of the mean value. Road direction also leads to a significant shrinkage of the range of ω scores for angular networks: while in symmetric context, the worst and best ω scores are respectively 0.51 and 0.32, which represents a difference of 0.19, all ω scores for non-symmetric angular street networks are comprised between 0.23 and 0.35, which is 0.07 less than the former range of values. These results, together with the similar analysis conducted in the previous section, put serious doubt on the relevance of the use of different angular fitness types and threshold values in the generation of angular street networks, at least for structural analysis.

4.2.4 SMALL-WORLDNESS ANALYSIS: CONCLUSIONS

As with the structural analyses conducted in section 4.1, investigation of the ω scores of all 120 street networks in this research allows to draw interesting trends regarding the effect of road direction on the street-based modeling of road networks. Globally, results seem to indicate that the ω metric helps make better sense of street networks. First of all, it allows for a clear, quantitative distinction between the orderliness of segmental networks and the other, more random-like, street networks. But also and most importantly, the ω metric makes possible comparative assessment of street network small-worldness: by comparing street networks one to another in terms of their respective ω scores, the coefficient provides the means to determine with both precision and accuracy their relative orderliness or randomness, as well as evaluate exactly how these street networks fare in terms of both global and local network efficiency.

As concerns the present research objectives, the ω metric allows for an effective assessment of the "road direction effect" on street network small-worldness. On that matter, results are indeed clear: taking into account road direction exacerbates the structural peculiarities of the different street networks. In light of the above-mentioned results and regardless of the neighbourhood, the road direction effect on street networks can be summed up as follows:

— Orderliness increases for all street networks (ω scores closer to -1 in non-symmetric context)
— Small-Worldness increase for odonymic and angular networks (ω scores closer to 0 in non-symmetric context)
— ω score homogenization for angular networks
— Near-optimal small-worldness for odonymic networks

From a strictly structural and small-world perspective, the "road direction effect" on the street-based modeling of road networks is thus quite real, hence the importance of taking it into account.

CONCLUSION

By gathering together all transportation routes ensuring the smooth circulation of pedestrians, cyclists, motorized vehicles (cars, bikes, trucks, ...) and goods, urban road networks constitute the largest urban infrastructures, accounting for more than a third of the total urban area of some american cities (Macdonald, 2011). On a structural level, road networks constitute essential support systems for urban life and development, as various morphological and structural aspects of modern cities are directly linked to the configuration of its road network, notably during critical phases of urban and demographic development (Borchert, 1961; Garrison and Marble, 1961; Kansky, 1963; Marshall, 2005).

But road networks are also more than city backbones: by allowing its users to live, work and interact in a multitude of different places, urban road networks are vested with a crucial feeding function, as essential to urban life as circulatory systems are to most animals. Thus, through their determining role in both the configuration of urban layouts and in the enabling of the different urban flows, road networks exert on urban space and life a determining organizational role.

Given the vital role of road networks for urban activity, issues and questions regarding their structural efficiency, more precisely their ability to maximize navigability throughout the network and to allow fast and simple access from everywhere to everywhere else, are of the utmost importance. In the present research, three different problems pertaining to road networks were considered, namely, their "legibility" (Lynch, 1992), which refers to the way road networks are perceived and represented by their users, the impact of road asymmetry on structural efficiency as well as the capacity to properly evaluate the latter. The first two areas of concern were dealt with through the generation of symmetric and non-symmetric street networks, while the last one was addressed through calculation and comparative analysis of ω scores for all street networks.

Altogether, these different initiatives have lead to the discovery of an intimate link between small-world networks and both symmetric and non-symmetric street networks. Regarding structural efficiency, such matching is very significative, as small-world networks represent optimal structures in terms of transportation, by ensuring a well-balanced tradeoff between local and global network efficiency.

Particularly interesting are the higher ω scores of non-symmetric street networks and the near-perfect small-worldness of non-symmetric odonymic street networks. From an urban design perspective, the fact that non-symmetric street networks have a higher small-worldness degree than their symmetric counterparts is very telling. Considering that the purpose of the partial orientation of road networks through the introduction of one-ways is to facilitate movement throughout the network, one must admit that, given the navigability of small-worlds and the higher small-worldness of non-symmetric network, this strategy achieves its goal rather efficiently. In fact, introducing one-ways might be the best way to optimize a network in terms of costs and benefits: by installing a few traffic posts at certain intersections, the network can be substantially reconfigured in very short time, with minimal work, manpower and expenses.

As regards to odonymic non-symmetric street networks, their near-perfect small-worldness scores are open to interpretation. Through lexicalization, roads acquire a conceptual dimension, which allow them to be referred to in everyday discourse and acquire meaning. Consequently, entire road networks become conceptual networks, which reduce all wayfinding activities and route directions to simple name sequences. Given this, the high small-worldness of non-symmetric odonymic networks means that the lexical/conceptual networks to which they correspond are extremely efficient from a structural point of view. Thus, while the geometry of road networks is far too complex to serve as proper navigational basis for all road network-related activities, their odonymic reduction is almost perfect in this respect. Such results are certainly evocative as regards to the wayfinding considerations presented in the introduction of this research. In order to highlight the potential contributions of this research to the study of wayfinding and cognition in general, a final digression seems here in order.

5.1 WAYFINDING, GRAPH KNOWLEDGE AND THE ART OF STREET LABELING

Originally, studies on the cognitive basis of wayfinding revolved around two different paradigms, a developmental one and an urbanistic one. The developmental approach has been first laid out by Piaget (Piaget and Inhelder, 1956). According to this model, the developmental sequence of spatial cognition in children is a geometric complexification process, according to which topological geometry is first mastered through familiarization with proximity, order separation and enclosure relations, followed by the later mastering of projective geometry and Euclidean, metric geometry.

While Piaget's work is important for spatial cognition, the first investigations focusing specifically on wayfinding were done by Kevin Lynch in *The Image of the City*. The main objective of this book "was to develop a method for the evaluation of city form based on the concept of imageability, and to offer principles for city design" (Raubal, 2008, p.1243). His research was based on interviews in which participants had to perform mental trips across their cities (either Boston, Los Angeles or Jersey City), describing or drawing the sequence of things they encounter along the way. This method, which quickly became a standard experimental procedure in wayfinding research, allowed Lynch to distinguish five types of structural elements that played a pivotal role in the mental representation of an urban environment: paths, edges, districts, nodes and landmarks.

> *Landmarks* are outstanding features (buildings, places) in a city. They serve as reference points to the observer. *Paths* are streets or lanes. *Nodes* are located along paths and form important physical barrier (e.g., rivers). *Dis-*

> *tricts* are areas in a city that have one common property (e.g., shopping areas, residential areas). Lynch discovered that an individual tends to use landmarks first in a new environment for the purpose of orientation. Gradually, the individuals add on to their knowledge until a cognitive map is constructed (Timpf, 1992, p.360).

Nestled between both paradigms and drawing heavily from the work of Shemyakin (1962), Siegel and White (1975) presented the a model of wayfinding in children based on the progression from landmark to route and survey knowledge.

> *Landmarks* are prominent features in the environment, which can be the location of an associated action or can serve as a beacon to aim toward. *Route knowledge* is usually defined by place-action associations, and it enables one to follow a known path from one location to another, perhaps encountering many landmarks along the way. *Survey knowledge* includes some information about the overall layout and how routes fit together - including knowledge of metric distances and angles - which gives one the ability to take novel shortcuts between locations" (Chrastil, 2013, p.208).

While the developmental aspect of Piaget's theory was based on cognitive maturation, the model of Siegel and White is based on knowledge acquisition: children become accustomed to a new environment by first acquiring knowledge of specific landmarks, then by drawing routes between landmarks:

> They suggested that by connecting several known landmarks and actions, children could establish all or part of a route. After several routes are known and correctly inter-related, children will have formed a "mini-map" - that is an accurate representation of the relationships between places and routes in a small area (...). With greater experience such "mini-maps" become integrated into a larger framework and children will then have achieved a "survey" representation of the environment (Blades, 1997, p.106).

Due to its focus on knowledge acquisition, Siegel and White's model also proved conducive to adult spatial cognition and wayfinding modelling (Evans et al., 1981). However, more recent experiments have shown that acquisition of spatial knowledge in both children and adults does not follow this rather rigid, three-part trajectory (McDonald and Pellegrino, 1993; Montello, 1998; Hoelscher et al., 2006; Devlin, 2012). First, manipulation of certain aspects of the environment has been shown to influence the type of spatial knowledge acquired (Taylor et al., 1999). Also, important individual differences in route and survey knowledge acquisition have been observed, thus putting into question the cognitive plausibility of the cognitive trajectory laid out by Siegel and White (Wolbers and Hegarty, 2010; Ishikawa and Montello, 2006). Finally, studies in neuroimaging have also exposed some shortcomings of the landmark/route/survey theoretical framework (Latini-Corazzini et al., 2010; Chrastil, 2013).

However, the main experimental argument against this model and that of Piaget relates to metric knowledge. Given that both models put the mastering of metric geometry at the top of the developmental chain, the poor performances observed for humans as well as the metric imperfection and inconsistency of their cognitive maps (Devlin, 2012) in this regard certainly put their validity or representativity into question. Indeed, since the pioneering experimentations made by Lynch (1992), it has often been remarked that "the nature of errors in peoples cognitive maps is most often metrical and only rarely topological" (Egenhofer and Mark, 1995, p.5), that spatial reasoning is based on topological structure (Stevens and Coupe, 1978) and that Euclidean, metric geometry "is not a good candidate for representing geographic information" (Egenhofer and Mark, 1995, p.5).

> In geographic space, topology is considered to be first-class information, whereas metric properties, such as distances and shapes, are used as refinements that are frequently less exactly captured. There is ample evidence that people organize geographic space such that topological information is retained fairly precisely (Egenhofer and Mark, 1995, p.9).

Experimentations made by Byrne (1979) are evocative in this regard, as they prove metric knowledge of urban network is incomplete and error-prone, even among subjects who have good knowledge of an area. In the first experiment, 80 undergraduates from the University of St.Andrews having stayed at least one year in that city were asked to estimate by ratio scaling the walking distances between pairs of locations in the city. The results showed that subjects' relative length estimates were strongly biased by the linearity and location of the routes analyzed, as "routes with several major bends are estimated longer than linear routes, and routes within the town centre are estimated longer than peripheral routes" (Byrne, 1979, p.152). According to Byrne, these biases can both be explained by postulating that subjects use mental representations which do not preserve or encode distance information and rely on the heuristic that the more locations (landmarks and turning points) are remembered on the route, the longer it must be. In a second experiment, 30 Cambridge residents were asked to estimate the angles between pairs of roads by drawing the configuration of roads at their junctions. Even though the real angles were either in the $60 - 70^o$ or $110 - 120^o$ range, all estimates differed little from 90^o, regardless of the true angular magnitude. These results, reminiscent of those obtained by Kevin Lynch, who found that many residents of Boston considered the Boston Common to be a square with five right-angled corners (Lynch, 1992), seem to indicate that in the absence of accurate angular information, the heuristic "roads intersect at right angles" prevails. In light of these experiments, "memory for urban geography" does not properly preserve or encode metric information (distance and angularity), but relies instead on error-prone heuristics. Given this, the author concludes that "vector maps", "in which the shapes, relative lengths of routes and the angles at which they join are routinely encoded and essential for the representation to be self-consistent" (Byrne, 1979, p.153), are less representative or reflective of actual mental representation of geographical information than network maps, which accurately and only preserve topological connectedness.

In light of all this, asserting that "topology matters, metric refines" (Egenhofer and Mark, 1995, p.5) might even be considered generous as regards to metric knowledge and competency. In order to explain this, appeal to the concept of relevance, as defined by Sperber and Wilson (Sperber and Wilson, 1986), might prove useful. According to these authors, human cognition is tuned to the maximization of relevance, as "humans draw from the perceptual input they receive just those bits of information that matter to them in some respect, and that relate to previous knowledge as well as current requirements, purposes, and needs" (Hirtle et al., 2011, p.74).

> When is an input relevant? Intuitively, an input (a sight, a sound, an utterance, a memory) is relevant to an individual when it connects with background information he has available to yield conclusions that matter to him: say, by answering a question he had in mind, improving his knowledge on a certain topic, settling a doubt, confirming a suspicion, or correcting a mistaken impression. In relevance-theoretic terms, an input is relevant to an individual when its processing in a context of available assumptions yields a POSITIVE COGNITIVE EFFECT. A positive cognitive effect is a worthwhile difference to the individual's representation of the world - a true conclusion, for example. False conclusions are not worth having. They are cognitive effects, but not positive ones (Wilson, 2002, p.608).

As regards to wayfinding activities, "relevance can be considered as the kind of spatial information that people need to know in order to effectively navigate" (Hirtle et al., 2011, p.75). In this sense, the poor performance observed for human participants in both metric information processing and recall is a sign that metric information is not relevant for most wayfinding tasks, topological geometry being here the only one that matters. In light of this, the landmark/route/survey knowledge model designed by Siegel and White "misses an important type of spatial knowledge - graph knowledge" (Chrastil, 2013, p. 210). This graph knowledge constitutes a sort of network-like cognitive map, emerging from route knowledge integration and serving as topological basis for survey knowledge acquisition. As already suggested in early experimental literature (Byrne, 1979), network or graph knowledge constitutes the cognitive basis of urban navigation, as "the knowledge that people rely on to travel around their neighbourhood or city" (Chrastil, 2013, p.210).

> For example, in a number of experiments participants have been asked to remember areas that they have learned very well, and then to recall a route between two locations (Maguire et al., 1997; Rosenbaum et al., 2004; Spiers and Maguire, 2006). While participants in those tasks are *planning a route*, they are not necessarily relying on *route knowledge*, which would only imply that participants knew one way to get from the start location to the goal,

> through a series of left and right turns, without requiring greater knowledge of the environment around them. Instead, those navigators likely could provide several possible routes, using their knowledge of the streets and roads in the environment to determine the most direct path. Indeed, the navigator might have never taken that particular route before. However, route planning is also not a test of survey knowledge, since accurate performance would not required knowledge of metric distances and angles between locations. Thus, an intermediate type of spatial knowledge is required, on in which navigators must use their knowledge of how the paths connect to find a suitable way to the goal (Chrastil, 2013, p. 210).

Research in street-based road network modeling, to which the present research subscribes, certainly supports this thesis: since the spatial cues which attract individuals' attention in urban environments should be those most relevant for navigation, topological relations between streets certainly seem to matter more than metric distances and directions between road segments. As regards to navigational relevance, however, the results presented in the previous chapter, by attesting to the amazing structural efficiency of odonymic street networks, do seem to carry additional explanatory potential. Indeed, for individuals who can read and communicate, odonymy represents an ideal navigational substitute to complex geometric calculations: by following street names instead of computing metric or even topological information, individuals are not only able to navigate with minimal cognitive effort, but also, given the high small-worldness scores of odonymic street networks, with maximal transportation efficiency. Also, given that odonymic knowledge "represents one of the fundamental structures on which we can base our assumptions about the spatial knowledge of others" (Tomko et al., 2008, p.51), the empirical success of route direction communication is testimony to the fact that "there is a large overlap in the the structures of our spatial knowledge, and that the knowledge of the prominent parts is common" (Tomko et al., 2008, p.51). But also and perhaps more importantly, given the small-worldness of odonymic networks, knowledge of these networks constitutes a "fast and frugal" way to ensure that navigational communication allows for maximally efficient transportation. Given all this, odonymy plays a huge role not only in compensating for the fragmentary and imperfect nature of the cognitive map of most individuals, but also in maintaining a healthy city life.

5.2 FURTHER DOWN THE ROAD TO DIRECTION: STREET NETWORKS AND THE STRUCTURAL DETERMINATION OF URBAN MOVEMENT

The objective of the present research has been to evaluate the impact of road asymmetry on the results obtained by street-based models in matters of network structure. However, the merits of street networks extend well beyond considerations of structural efficiency, as interesting results were also obtained regarding the structural determination of traffic flows. As an invitation to further research, the following section provides a short description of the benefits and shortcomings of street-based traffic modeling.

Traditionally, traffic modeling starts from a rather simple cognitive or behavioral assumption, rather common in both transportation and telecommunication studies, which is that, given the prevailing traffic conditions, road network users choose the travel route they perceive as being the shortest (Kohl, 1841). Applied to all travelers, this modeling principle implies that "drivers cannot reduce theory journey times by unilaterally choosing another route" (Correa and Stier-Moses, 2011, p.1), thus resulting in an overall traffic flow distribution that has been very early in the literature qualified as an equilibrium state (Knight, 1924). This notion of traffic flow equilibrium as well as its underlying principle of total travel cost minimization have received their proper, definitive formulation in Wardrop's first principle, which states that "the journey times on all the routes actually used are equal, and less that those which would be experienced by a single vehicle on any unused route" (Wardrop, 1952, p.345). Formalized in 1956 (Beckmann et al., 1956), Wardrop's first principle has been considered since its formulation as a sound behavioral principle for traffic modeling, and Wardrop equilibrium models have been used since then for predictive purposes, notably for the simulation of commuting in real-life networks (Sheffi, 1985) and of traffic distributions over alternate routes in cases of congestion (Florian, 1999).

> These models have been and are still used today to evaluate alternative future scenarios and decide a route of actions. Typical examples include allocation of investment for capacity expansion when building roads and bridges, optimizing the value of tolls, and making policy decisions (Correa

and Stier-Moses, 2011, p.1).

Over the years, transportation planners and engineers have experimented with different cost minimization scenarios: besides travel time and distance, turns, left turns, actual or perceived cost, lights and stop signs are among the most prominent examples (Golledge and Gärling, 2004). Recent research has however called in question the cognitive plausibility of travel optimization, at the core of Wardrop's first principle and Wardrop equilibrium models. First, studies in mode choice (Exel and Rietveld, 2010) and route choice (Levinson et al., 2006, 2004; Zhang et al., 2005) have shown that the influence of travel time on travel behaviour is changing and that even the perception of travel time is often far from precise (Parthasarathi et al., 2011). Moreover, GPS analysis of more than 1500 taxi trips made in the Japanese city of Nagoya has shown that a high proportion of trips do not satisfy the principle of distance minimization (Morikawa et al., 2005).

Given these cognitive adequacy and experimental validation issues, the question arises as to whether the "cognitive hurdles" regarding road choice and behaviour have not been set too high by Wardrop equilibrium models. In this regard, recent research on the impact of street networks on traffic flows are evocative: by advocating a "top-down" modeling approach, free of any individual or cognitive considerations, instead of simulating traffic flows "from the bottom up", that is, on the basis of individual preferences and behaviors, street-based models have obtained impressive results. Such top-down studies are centered on the principle of 'natural movement', which refers to the proportion of total urban movement that is determined by urban structure itself - street networks in the present case (Hillier et al., 1993).

> In a large and well developed urban grid people move in lines, but start and finish everywhere. We cannot easily conceive of an urban structure as complex as the city in terms of specific generators and attractors, or even origins and destinations, but we do not need to because the city is a structure in which origins and destinations tend to be diffused everywhere, though with obvious biases toward higher density areas and major traffic interchanges.

> So movement tends to be broadly from everywhere to everywhere else. To the extent that this is the case in most cities, the structure of the grid itself accounts for much of the variation in movement densities (Hillier, 1996, p. 120).

Many studies on street-based modeling have confirmed the high correlation rate between street centrality values and movement density values: the more integrated (central) a street in a street networks, the more it is used by travelers and thus the higher its movement density (Hillier and Hanson, 1984; Hillier, 1996; Hillier et al., 1993; Penn et al., 1998; Turner, 2007). More precisely, GPS trip analysis of about 50 taxicabs from the city of Gävle and for a period of one week revealed that over %80 of overall traffic occurred in the top 20% of well-connected odonymic or angular streets (those that have the highest number of neighbouring streets), thus satisfying the Pareto principle (or 80-20 rule) (Jiang, 2009b).

Closer to the present research, the PageRank and Weighted PageRank values for the odonymic and angular streets of the three London neighbourhoods of Barnsbury, Clerkenwell and Kensington, have been shown to correlate as high as 70% with observed traffic flow (Jiang and Jia, 2011). This same study, through comparison of purposive and random movements in the street networks of the same neighbourhood, has shown that both types of movement result in similar movement patterns, which seems to support the thesis that, contrary to what has always been postulated in traffic modeling, "higher cognitive abilities are not required in the formation of movement patterns at a collective level" (Jiang and Jia, 2011, p.8): movement in road networks would thus be mainly determined "through the interaction between moving agents and the underlying street networks and have little to do with agents' cognition" (Jiang and Jia, 2011, p.8).

However, as with the street-based modeling in general, these impressive results are somehow hampered by the fact that road direction is not taken into account in the construction of street paths. Thus, as a matter for future research and in order to properly evaluate the soundness of street-based models, an assessment of the effect of road direction on the structural determination of urban movement by street-based models might seem in order.

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